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1. $f(x) = \ln x - (\ln x)^2$

a) $\ln x \, (\underbrace{1 - \ln x}) = 0$

$x = 1 \qquad x = e$

$f'(x) = \frac{1}{x} - 2\ln x \cdot \frac{1}{x} = \frac{1}{x}(1 - 2\ln x)$

b) $\frac{1}{x}(\underbrace{1 - 2\ln x}) = 0$

$x = \sqrt{e}$

$y = \frac{1}{4}$ $\qquad A(\sqrt{e} \mid \frac{1}{4})$

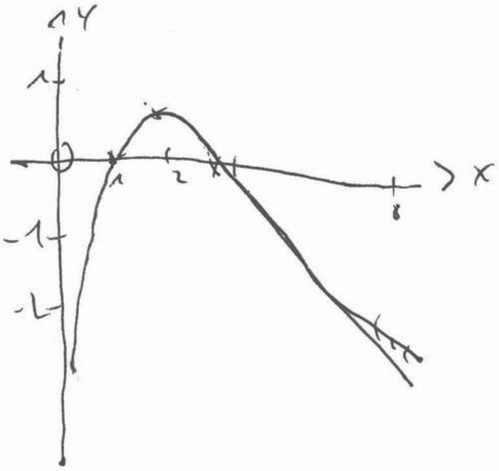
c) $f''(x) = -\frac{1}{x^2}(1 - 2\ln x) + \frac{1}{x}\left(-\frac{2}{x}\right)$

$= + \frac{1}{x^2}(-3 + 2\ln x) = 0$

$x = e^{\frac{3}{2}}$

$y = -\frac{3}{4}$ $\qquad B(\sqrt{e^3} \mid -\frac{3}{4})$

d)



e) $F(x) = -3x + 3x\ln x - x(\ln x)^2$

$F'(x) = -3 + 3\ln x + 3 - (\ln x)^2 - 2\ln x$

$= \ln x - (\ln x)^2 = f(x)$ ✓

$$\int_1^e f(x)\,dx = \left[-3x + 3x\ln x - x(\ln x)^2\right]_1^e$$

$= 3 - e$

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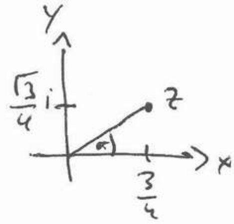
2. $w = f(z) = \frac{3+\sqrt{3}i}{4} \cdot z$

$z_0 = 4 \; ; \; z_{n+1} = f(z_n)$

a) $z = \frac{3+\sqrt{3}i}{4} = \frac{3}{4} + \frac{\sqrt{3}}{4} i$



$= \frac{1}{2}\sqrt{3}\, e^{\frac{\pi}{6} i}$

$= \underline{\underline{\frac{1}{2}\sqrt{3}\, cis(30^\circ)}}$

$\tan\alpha = \frac{\sqrt{3}}{3}$ $\qquad |z| = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2}$

$\alpha = 30^\circ$ $\qquad = \frac{1}{2}\sqrt{3}$

Drehstreckung : Winkel 30°
Faktor $\frac{1}{2}\sqrt{3}$

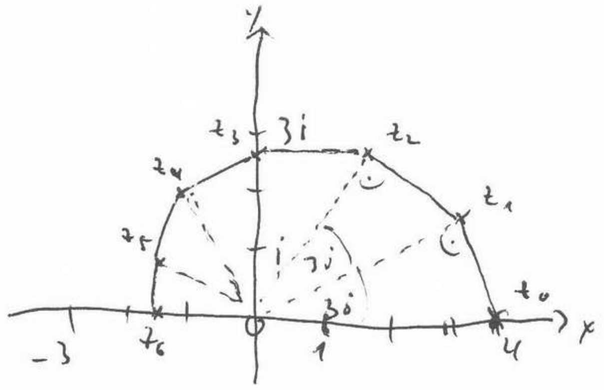
b) $z_n = \left(\frac{3+\sqrt{3}i}{4}\right)^n z_0 = \left(\frac{3+\sqrt{3}i}{4}\right)^n \cdot 4$

$z_2 = \left(\frac{3+\sqrt{3}i}{4}\right)^2 \cdot 4 = \frac{9 + 6\sqrt{3}i - 3}{16} 4 = \underline{\underline{\frac{3}{2} + \frac{3}{2}\sqrt{3}i}}$

c) $z_n = \left(\frac{3+\sqrt{3}i}{4}\right)^n \cdot 4 = \left(\frac{1}{2}\sqrt{3}\, cis(30^\circ)\right)^n \cdot 4 = 4\left(\frac{\sqrt{3}}{2}\right)^n \cdot cis(30^\circ \cdot n)$

$z_0 = 4 \; ; \; z_1 = 2\sqrt{3}\, cis(30^\circ) \; ; \; z_2 = 3\, cis(60^\circ) \; ; \; z_3 = \frac{3}{2}\sqrt{3}\, cis(90^\circ)$

$z_4 = \frac{9}{4} cis(120^\circ) \; ; \; z_5 = \frac{9}{8}\sqrt{3}\, cis(150^\circ) \; ; \; z_6 = \frac{27}{16} cis(180^\circ)$



d) $\left|\frac{z_{n+1}}{z_n}\right| = \frac{\sqrt{3}}{2}$; Winkel 30°, also ist $\Delta O z_n z_{n+1}$ rechtwinklig

somit ~~zn zn~~ $\overline{z_{n+1} z_n} = |z_n| \cdot \sin 30^\circ = \frac{1}{2} \cdot |z_n|$

also $|z_n| = \left(\frac{1}{2}\right)^n \cdot 4$

$\underline{\underline{L}} = \lim_{n\to\infty} \sum_{i=0}^{n} |z_n| = \lim_{n\to\infty} \sum_{i=0}^{n} \left(\frac{1}{2}\right)^n \cdot 4 = \lim_{n\to\infty} \frac{1 - \left(\frac{1}{2}\right)^{n+1}}{1 - \frac{1}{2}} \cdot 4 = \underline{8}$

e) $\frac{z_m}{z_0} = \frac{z_1}{z_m} \implies z_m^2 = z_1 \cdot z_0 = 12 + 4\sqrt{3}i = 8\sqrt{3}\, cis(30^\circ)$

$Re(z_m) > 0$

$z_m = \sqrt{8\sqrt{3}} \cdot cis\left(\frac{30^\circ + 2k \cdot 180^\circ}{2}\right) \quad k = 0,1$

$z_{m,0} = 2\sqrt{2\sqrt{3}} \cdot cis(15^\circ) \; ; \; z_{m,1} = 2\sqrt{2\sqrt{3}} \cdot cis(195^\circ)$

$\underline{\underline{z_m = 2\sqrt{2\sqrt{3}}\, cis(15^\circ)}}$

3. $A(6|5|2)$ $B(6|6|7)$ $Q(2|17|12)$

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a) $\underline{\underline{A_{\Delta ABQ}}} = \frac{1}{2}|\vec{AB} \times \vec{AQ}| = \frac{1}{2}\left|\begin{pmatrix}0\\1\\5\end{pmatrix} \times \begin{pmatrix}-4\\12\\10\end{pmatrix}\right| = \frac{1}{2}\left|\begin{pmatrix}-50\\-20\\4\end{pmatrix}\right| = \underline{\underline{27}}$

b) $E_{ABQ}: \quad -50x - 20y + 4z + d = 0$

$A(6|5|2): \quad d = 192$

HNF $\quad \dfrac{-50x - 20y + 4z + 192}{54} = 0$

$(0|0|0): \quad \dfrac{192}{54} = \underline{\underline{\dfrac{32}{9}}}$

c) $X = (0|y|z)$: I $|\vec{AX}| = |\vec{BX}|$: $6^2 + (y-5)^2 + (z-2)^2 = 6^2 + (y-6)^2 + (z-7)^2$

II. $|\vec{AX}| = |\vec{CX}|$: " $= 9 + (y+3)^2 + (z-7)^2$

$y = 3 \quad z = 5$

$\underline{\underline{X(0|3|5)}}$ $\quad \underline{\underline{u = |\vec{AX}| = 7}}$

d) $g: \vec{X} = \underset{P}{\begin{pmatrix}15\\9\\6\end{pmatrix}} + t\underset{\vec{v}}{\begin{pmatrix}4\\3\\1\end{pmatrix}}$ $\quad \vec{AP} = \begin{pmatrix}9\\4\\4\end{pmatrix}$ $\quad \cos\alpha = \dfrac{\vec{AP} \cdot \vec{v}}{AP \cdot v} = \dfrac{52}{\sqrt{113} \cdot \sqrt{26}}$

$\alpha = 16{,}39°$

$\underline{\underline{d(A;g)}} = AP \cdot \sin\alpha = \underline{\underline{3}}$

$\sin^2\alpha = 1 - \cos^2\alpha = \frac{9}{113}$; $\underline{\underline{d(A;g)}} = \sqrt{113} \cdot \frac{3}{\sqrt{113}} = \underline{\underline{3}}$

e) $K: \quad x^2 + y^2 + z^2 = 518 \quad \cap g$

$(4t + 15)^2 + (3t + 9)^2 + (t+6)^2 = 518$

$t_1 = \frac{11}{13}$

$t_2 = -8$

Alle Punkte mit $t \in \,]-\infty; -8[\, \cup \,]\frac{11}{13}; \infty[$

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4.

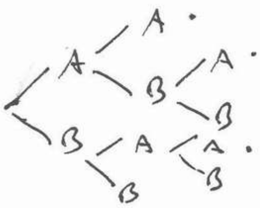
a) $P(\text{Anna einen Punkt in 3 Spielen}) = 3 \cdot \frac{1}{3} \cdot \left(\frac{2}{3}\right)^2 = \frac{4}{9} = 44{,}4\%$

b) $P(\text{Anna höchstens 4 Punkte in 12 Spielen})$

$$= \sum_{i=0}^{4} \binom{12}{i} \left(\frac{1}{3}\right)^i \left(\frac{2}{3}\right)^{12-i} = \left(\frac{2}{3}\right)^{12} + 12 \cdot \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{11} + \frac{12!}{2!\,10!}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{10} + \frac{12!}{3!\,9!}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^9 + \frac{12!}{4!\,8!}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^8$$

$$= 63{,}2\%$$

c) $P(\text{beendet, wer zuerst 2 Punkte}) = \left(\frac{1}{3}\right)^2 + 2 \cdot \left(\frac{1}{3}\right)^2 \cdot \frac{2}{3} = \frac{7}{27} = 25{,}9\%$



d) $P(\text{In } n \text{ Versuchen mind. 2 Punkte}) > 0{,}99$

$1 - P(n \text{ 0 od. 1 Punkt}) > 0{,}99$

$1 - \left[\left(\frac{2}{3}\right)^n + n \cdot \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{n-1}\right] > 0{,}99$

$\left(\frac{2}{3}\right)^n + n\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{n-1} < 0{,}01$

$n \geq 17$

e)

$$E = 18 \cdot P(\text{Summe der zwei Augen} > 8) + 9\,\overbrace{P(\text{Summe zwei Augen} \leq 8)}^{1-P(>8)}$$

$$= 9 + 9\,P(>8) = 11{,}5$$

$P(>8)$: $\frac{1}{36} \cdot (1 + 2 + 3 + 4) = \frac{5}{18}$

Summe:	12	11	10	9
	6,6	6,5	6,4	6,3
		5,6	4,6	3,6
			5,5	5,4
				4,5

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5 a) $f(x) = \frac{5}{2} \sin\left(\frac{x}{2}\right) + \frac{1}{2}$

Streckung Y

Stauchung X

Verschiebung Y

b) $\sin x = \cos x$

$x = 45°$

$f'(x) = \cos x$

$g'(x) = -\sin x$

$f'(45°) = \frac{1}{2}\sqrt{2} = \tan\alpha \Rightarrow \alpha = 35{,}26°$

$g'(45°) = -\frac{1}{2}\sqrt{2} = \tan\beta \Rightarrow \beta = -35{,}26$

Schnitt : $70{,}5°$

$\underline{\underline{= 71°}}$

c) $(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^n b^{n-i}$

$(1+1)^n = 2^n = \sum_{i=0}^{n} \binom{n}{i} 1^n 1^{n-i}$

$\underline{\underline{2^n = \sum_{i=0}^{n} \binom{n}{i}}} = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n}$

d) $A(2|3|3)$ $B(-3|-2|2)$ $C(0|0|z)$

$\vec{AC} \cdot \vec{BC} = 0$

$\begin{pmatrix} -2 \\ -3 \\ z-3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ z-2 \end{pmatrix} = 0$

$-6 - 6 + (z-3)(z-2) = 0$

$z_1 = 6$

$z_2 = -1$

$\underline{C_1(0|0|6)}$

$\underline{C_2(0|0|-1)}$

e) $\lim_{x \to \frac{\pi}{3}} \frac{\cos x - \frac{1}{2}}{e^{3x-\pi} - 1} = \lim_{x \to \frac{\pi}{3}} \frac{-\sin x}{3e^{3x-\pi}} = \frac{-\frac{1}{2}\sqrt{3}}{3} = \underline{\underline{-\frac{1}{6}\sqrt{3}}}$