ENIF15

1. $f_p(x) = (p-x)e^x$

a) $f_p'(x) = -e^x + (p-x)e^x = (p-1-x)e^x$

$f_p''(x) = (p-2-x)e^x$

$f' = 0$

$\underline{x = p-1}$ $\quad f''(p-1) = -\underbrace{e^x}_{>0} \neq 0$ für alle x

also Hochpunkt

$\underline{HP(p-1 \mid e^{p-1})}$

b) $f''(x) = 0$

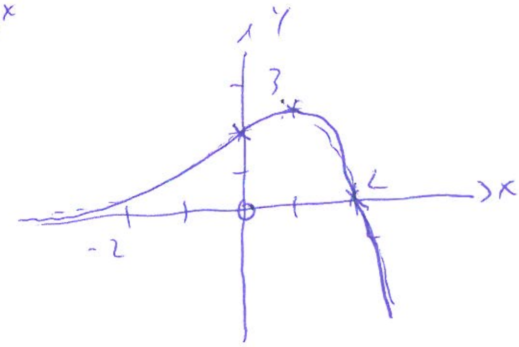
$x = p-2$ einfache NST, also $\underline{WP(p-2 \mid 2e^{p-2})}$

also VZW

c) $x = p-2$

$y = 2e^{p-2} = 2e^x$: Ortskurve WP : $\underline{y = 2e^x}$

d) $f_2(x) = (2-x)e^x$



e) $f_p^{(k)}(x) = (p-k-x)e^x$

$f_p^{(-1)}(x) = F_p(x) = (p+1-x)e^x$

$F_p'(x) = -e^x + (p+1-x)e^x$

$= (p-x)e^x = f_p(x)$ ✓

LENIHS

2. a) $(2+3i)^2 - \overline{(5-2i)^2}$

$= 4 + 12i - 9 - \overline{(25 - 20i - 4)}$

$= -5 + 12i - 21 + 20i$

$= \underline{-26 + 8i} = \underline{-26 - 8i}$

b) $z^3 = (-27)i = 27(-i)$

$z = -3 \cdot \left(\cos\left(\frac{\pi}{6} + k \cdot \frac{2\pi}{3}\right) + i \sin\left(\frac{\pi}{6} + k\frac{2\pi}{3}\right)\right) \qquad k = 0;1;2$

c) $w = f(z) = (3+4i) \cdot z + 2 - 6i$

$\Leftrightarrow \quad z = (3+4i)w + 2 - 6i$

$w = \frac{z - 2 + 6i}{3+4i} = \frac{1}{3+4i} z + \frac{6i}{3+4i}$

$= \frac{3-4i}{9+16} z + \frac{6i(3-4i)}{9+16}$

$= \underbrace{\left(\frac{3}{5} - \frac{4}{5}i\right)}_{z_1} z + \underbrace{\left(\frac{24}{5} + \frac{18}{5}i\right)}_{z_2}$

d) $|(a+ib)^2| = |a^2 - b^2 + 2abi| = \sqrt{(a^2 - b^2 + 2abi)(a^2 - b^2 - 2abi)} =$

$|a+ib|^2 = \sqrt{(a+ib)(a-ib)}^2 = \underline{\underline{a^2+b^2}}$

$= \sqrt{(a^2-b^2)^2 + 4a^2b^2}$

$= \sqrt{a^4 - 2a^2b^2 + b^4 + 4a^2b^2}$

$= \sqrt{a^4 + 2a^2b^2 + b^4}$

$= \sqrt{(a^2+b^2)^2}$

$= \underline{\underline{a^2+b^2}}$

e) $z_0 = 3 + 4i$

$z_1 = (3+4i) \cdot (0{,}15 + 0{,}2i)$

$z_2 = \quad " \quad " \quad ^2 \qquad q = (0{,}15 + 0{,}2i)$

$|q| = 0{,}25$

$z_n = z_0 \cdot q^n$

$\underline{\underline{S_n}} = \sum_{n=0}^{\infty} |z_n| \qquad |z_n| = |z_0| \cdot |q|^n$

$= 5 \cdot \left(\frac{1}{5}\right)^n$

$= \sum_{n=0}^{\infty} 5 \cdot \left(\frac{1}{5}\right)^n$

$= 5 \cdot \frac{1}{1-q} = 5 \cdot \frac{1}{1-\frac{1}{5}} = \underline{\underline{\frac{25}{4}}}$

ENIF15

3. $E: x-2y-2z+2=0$

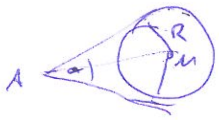
$M(-4|9|8) \quad R=15$

a) x-Achse $(y=z=0)$ $\quad x=-2$

y $\quad$. $\quad$ .. $\quad y=1$

$z=1$

b) $A(6|-19|12)$



$\sin(\varphi/2) = \frac{R}{AM} = \frac{15}{30} = \frac{1}{2}$

$\underline{\varphi = 60°}$

c) $HNF(E) = \frac{x-2y-2z+2}{3} = 0$

$M(-4|9|8): \left| \frac{-4-2\cdot 9-2\cdot 8+2}{3} \right| = \underline{12 = d(M;E)}$

$\underline{r} = \sqrt{R^2-d^2} = \sqrt{15^2-12^2} = \underline{9}$

d) $P(x_0|-2|6): \quad x_0-2\cdot(-2)-2\cdot 6+2=0$

$\underline{x_0 = 14}$

$P(14|-2|6): \quad \vec{PM} = \begin{pmatrix} -18 \\ 11 \\ 2 \end{pmatrix} = \vec{n}$

$T: -18x+11y+2z+d=0$

$d=262$

$\underline{T: -18x+11y+2z+262=0}$

e) $MM^*: \vec{x} = \begin{pmatrix} -4 \\ 9 \\ 8 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \wedge E: \quad 9t-36=0$

$\underline{t=4}$

$\underline{\vec{r}_{M^*} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$

$\vec{PM^*} = \begin{pmatrix} 14 \\ 3 \\ 6 \end{pmatrix} \qquad \vec{PM^*} \times \vec{n} = \begin{pmatrix} \cancel{16} \\ \cancel{34} \\ -267 \end{pmatrix} \begin{pmatrix} 6 \\ 34 \\ -31 \end{pmatrix}$

$\underline{t: \vec{x} = \begin{pmatrix} 14 \\ -2 \\ 6 \end{pmatrix} + t\begin{pmatrix} 6 \\ 34 \\ -31 \end{pmatrix}}$

4. 3w | 7s LEN | F15

10 Z. m. Z.

a) $W(3w, 7s) = \binom{10}{3} \cdot 0{,}3^3 \cdot 0{,}7^7 = 26{,}7\%$

b) $WS(\text{w höchstens 2mal}) = \binom{10}{0} 0{,}3^0 \cdot 0{,}7^{10} + \binom{10}{1} 0{,}3^1 \cdot 0{,}7^9 + \binom{10}{2} 0{,}3^2 0{,}7^8$

$= \underline{38{,}3\%}$

c) $W(\text{in N Zügen mind. ein weiße}) > 99\%$

$1 - W(\text{in N Zügen kein weiß}) > 0{,}99$

$1 - (0{,}7)^n > 0{,}99$

$0{,}7^n < 0{,}01$

$n > \log_{0{,}7} 0{,}01 = 12{,}9$

ab 13 Zügen

d) 2 Kugeln ohne Z.

$G(0w) = 1,-$
$G(1w) = 2,-$
$G(2w) = 3,-$

$Z = p(0w) \cdot G(0w) + p(1w) G(1w) + p(2w) G(2w)$

$= \frac{7 \cdot 6}{10 \cdot 9} \cdot 1,- + \frac{3 \cdot 7}{10 \cdot 9} \cdot 2 \cdot 2,- + \frac{3 \cdot 2}{10 \cdot 9} \cdot 3,-$

$= \underline{1{,}6,-}$

e) 10 mal ohne Z.

$P(\text{1 und letzte schwarz}) = \frac{\binom{8}{3}}{\binom{10}{3}} = \frac{7}{15} = \underline{46{,}7\%}$

$\binom{10}{3}$ alle Mgl.

$\binom{8}{3}$ Mgl. bei mit 1. und 10. schwarz

ENIF15

5 a) $A(5|2|1)$ $B(8|6|6)$ $C(9|6|7)$

$\overline{AB} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$ $\overline{AC} = \begin{pmatrix} 4 \\ 4 \\ 6 \end{pmatrix}$

$\underline{A} = \frac{1}{2} \left| \overline{AB} \times \overline{AC} \right| = \frac{1}{2} \left| \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix} \right| = \underline{3}$

b) Symmetrie: Schnittl Parabel bei $x = 3$

$y = a(x-3)^2 + b$

$y' = 2a(x-3)$

I $y'(1) = \frac{3}{1}$ (m_{AB})

II. $y(1) = 3$

I. $2a(1-3) = 3$

$\underline{a = -\frac{3}{4}}$

II. $-\frac{3}{4}(1-3)^2 + b = 3$

$\underline{b = 6}$

$\underline{y = -\frac{3}{4}(x-3)^2 + 6}$

c) $z = 2^{3n} + 13 \mid 7$?

$n = 1$: $z = 2^3 + 13 = 21 \mid 7$ ✓ Induktionsanfang

Sei: $2^{3n+1} + 13 \mid 7$ (n)

$(n+1)$: $2^{3(n+1)+1} + 13 = 2^{3n+1+3} + 13 = \underbrace{8}_{1+7} \cdot 2^{3n+1} + 13$

$= \underbrace{\underbrace{2^{3n+1} + 13}_{\mid 7 \text{ nach V.}} + \underbrace{7 \cdot 2^{3n+1}}_{\mid 7 \text{ da 7 Faktor}}}_{\mid 7}$ ✓ Induktionsschluss

d) $\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $M = \begin{pmatrix} -1 & 2 \\ -6 & 6 \end{pmatrix}$

$M\vec{v} = \lambda \vec{v}$

$(M - 1\lambda)\vec{v} = 0$

$\det(M - 1\lambda) = 0$

$(-1-\lambda) \cdot (6-\lambda) + 12 = 0$

$\underline{\lambda_1 = 2}$

$\underline{\lambda_2 = 3}$

λ_1: $-3x + 2y = 0$

$y = \frac{3}{2}x$

$\underline{\vec{v}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \vec{b}}$

λ_2: $-4x + 2y = 0$

$y = 2x$

$\underline{\vec{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \vec{a}}$

e) $\int_0^a x(a-x)\,dx = 36$

$\left[\frac{a}{2}x^2 - \frac{1}{3}x^3\right]_0^a = 36$

$\frac{a^3}{2} - \frac{a^3}{3} = 36$

$\underline{a = 6}$