F16
EN

1. $f_p(x) = x^3 + p(x^2 + x) + 2$

a) $p \cdot \underbrace{x}_{0} \cdot \underbrace{(x+1)}_{-1} = 0$

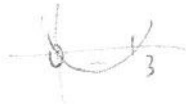
$\underline{x = 0}$ und $\underline{x = -1}$

b) $f'_p(x) = 3x^2 + p(2x+1) = 0$

$D = 4p^2 - 4 \cdot 3 \cdot p$

$p(p-3) < 0$

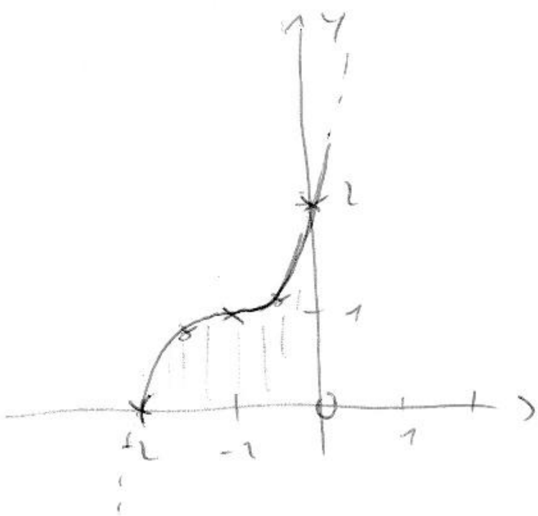
$\underline{p \in \,]0;3[}$



c) $f''_p(x) = 6x + 2p = 0$

$\underline{x = -\frac{1}{3}p}$ einfache Lsg.

d) $p = 3$



WP$(-1|1)$ $f'(-1) = 0$

Terrassenpunkt

e) $\underline{A} = \int_{-2}^{0} f(x)\,dx = \left[\frac{1}{4}x^4 + x^3 + \frac{3}{2}x^2 + 2x\right]_{-2}^{0} = \underline{2}$

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2. $u = 3+4i \qquad v = 216\,\text{cis}(30°)$

a) $\tan\alpha = \frac{4}{3} \rightarrow \alpha = 53{,}13°$

$|u| = 5$

$\underline{u = 5 \cdot \text{cis}(53{,}13°)}$

$x = 216 \cdot \cos 30° = 108\sqrt{3}$

$y = 216 \cdot \sin 30° = 108$

$\underline{v = 108\sqrt{3} + i\,108}$

b) $z^3 = v$

$\sqrt[3]{216} = 6$

$z_1 = 6 \cdot \text{cis}(10°)$

$z_2 = 6 \cdot \text{cis}(10° + \frac{2\pi}{3})$

$z_3 = 6 \cdot \text{cis}(10° + \frac{4\pi}{3})$

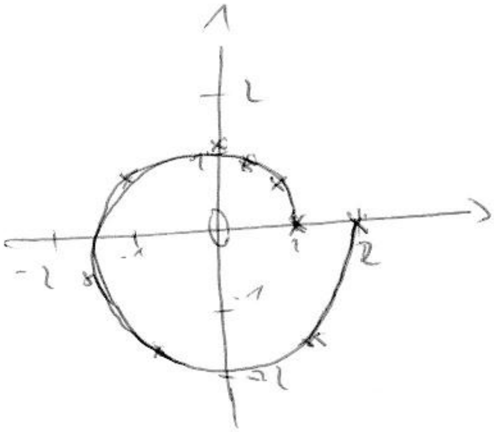
c) $1 + q + q^2 + \ldots = \sum_{i=0}^{\infty} q^i \qquad q = \frac{2}{3} + \frac{1}{4}i$

$|q|^2 = \frac{73}{144} < 1$ also $|q| < 1$

$= \frac{1}{1 - \left(\frac{2}{3} + \frac{1}{4}i\right)}$

$= \underline{\frac{48}{25} + \frac{36}{25}i}$

d) $z(\varphi) = 2^{\left(\frac{\varphi}{2\pi}\right)} \text{cis}(\varphi)$



e) $\left(\overline{1+2i}\right)^4 = (1-2i)^4$

$= 1^4 - 4 \cdot 1^3(2i) + 6 \cdot 1^2(2i)^2 - 4 \cdot 1^3(2i)^3 + (2i)^4$

$= 1 - 8i - 24 + 32i + 16$

$= \underline{-7 + 24i}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

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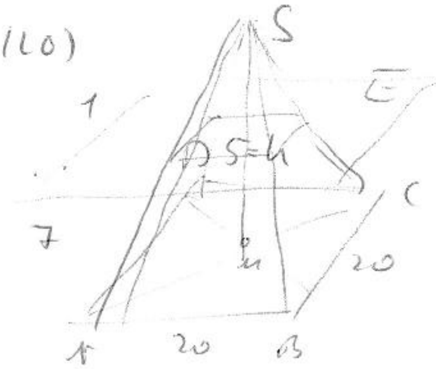


3. $A(12|-8|4)$ $B(12|12|4)$ $C(24|12|20)$

a) $\vec{r}_M = \frac{1}{2}(\vec{r}_A + \vec{r}_C) = \begin{pmatrix} 18 \\ 2 \\ 12 \end{pmatrix}$

$\underline{M(18|2|12)}$

$\vec{r}_D = \vec{r}_A + \overrightarrow{BC} = \begin{pmatrix} 24 \\ -8 \\ 20 \end{pmatrix}$

$\underline{D(24|-8|20)}$

b) $\vec{n} = \overrightarrow{AB} \times \overrightarrow{BC} = \cancel{320} \begin{pmatrix} 320 \\ 0 \\ -240 \end{pmatrix} \sim \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ $|\vec{n}| = 5$

$\vec{r}_{S_{1,2}} = \vec{r}_M \pm \vec{n}$ $\underline{S_1(22|2|9)}$

$\underline{S_2(14|2|15)}$ $\leftarrow S$

c) $V_{Spitze} = \frac{1}{8} V_{Gesamt}$ $h \to \frac{1}{2} h$

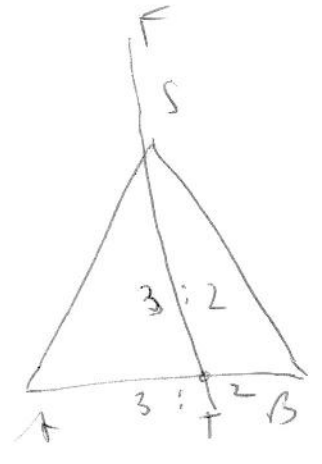
$V \to \frac{1}{8} V$

$\vec{r}_{M'} = \vec{r}_M + \frac{1}{2}\vec{n}$ $\to$ $\underline{M'(16|2|\frac{27}{2})}$

$E: \quad 4x + 0y - 3z + d = 0$

$M' \quad 4 \cdot 16 \quad - 3 \cdot \frac{27}{2} + d = 0$

$\underline{E: \; 4x - 3z - \frac{47}{2} = 0}$



d) $S \in F$; $F \parallel BC$

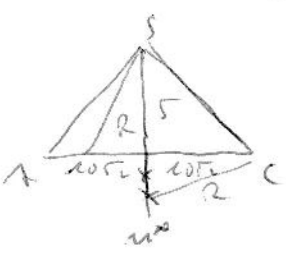
$\overrightarrow{AB} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}$

$|\overrightarrow{AB}| = 20$

$\frac{3}{5} - \frac{1}{2} = \frac{1}{10} AB$ $\frac{1}{10} AB = 2$

$A_{F \, in \, Pyr} = \frac{1}{2} BC \cdot \sqrt{29} = \frac{1}{2} 20 \cdot \sqrt{29} = \underline{10\sqrt{29}}$

e)



$(5 + x)^2 = (10\sqrt{2})^2 + x^2$

$x = \frac{35}{2}$

$\vec{r}_{M^*} = \vec{r}_M + \frac{35}{2} \cdot \frac{1}{5} \cdot \vec{n}$

$\underline{M^*(32|2|\frac{3}{2})}$

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4) 15 Münzen, 5 gewicht

a) 3 Z.o.Z.

$P(\text{mind. ein ge.}) = 1 - P(\text{kein gezinkt})$

$= 1 - \frac{10 \cdot 9 \cdot 8}{15 \cdot 14 \cdot 13} = \underline{73{,}63\%}$

b) $P(\text{in } n \text{ Zügen mind. ein gez.}) > 95\%$

$1 - P(n \text{ keine ge.}) > 0{,}95$

$\left(\frac{2}{3}\right)^n < 0{,}05$

$n > \log_{\frac{2}{3}} 0{,}05 = 7{,}388$

ab 8 Züge

c) Münze fair: $\underline{p(KZ) = p(ZK) = \frac{1}{4}}$

bei unfair: $p(K) = p \rightarrow p(Z) = 1-p$

$\underline{p(KZ)} = p(1-p) = \underline{p(ZK)} = (1-p)p$

d) $p(K) = 60\%$ Gewinn: ZZ: 30, KK: 10, ZK/KZ: x

$E = 18$

$18 = 0{,}4^2 \cdot 30 + 0{,}6^2 \cdot 10 + 2 \cdot 0{,}4 \cdot 0{,}6 \cdot x$

$\underline{x = 20}$

e) $p_1(K) = 70\%$, $p_2(K) = 40\%$

$1 - Z_1 Z_2 = 82\%$

$\underline{\frac{70}{82} = 85{,}4\%}$

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5. a) $3^n - 3 = 3(3^{n-1} - 1) = \underbrace{3\cdot\overbrace{(3-1)}^{2}}_{6}\cdot(\ldots)$

$a^n - b^n = (a-b)(a^{n-1} \ldots)$

$3^n - 3 = 6\cdot(\ldots)$ also durch 6 teilbar

b) Periode $7-3=4$: $\frac{2\pi}{4}$
3 nach rechts $x-3$
3 nach oben $y+3$

$A\cdot \sin\left[\frac{2\pi}{4}(x-3)\right] + 3 = f(x)$

Steigung AB: $1 = f'(3)$

$1 = A\cdot\frac{2\pi}{4}\cos\left(\frac{2\pi}{4}(x-3)\right)\Big|_{x=3}$

$1 = A\cdot\frac{2\pi}{4}$

$A = \frac{2}{\pi}$

$f(x) = \frac{2}{\pi}\sin\left[\frac{\pi}{2}(x-3)\right] + 3$

c) $\vec{AB} = \begin{pmatrix}5\\3\\2\end{pmatrix}$ $\quad \vec{DC} = \begin{pmatrix}5\\3\\2\end{pmatrix}$ $\quad \vec{AB} = \vec{DC}$ also Parall.

d) $z = 2016^{2016} \quad | \lg$

$\lg z = 2016 \lg 2016 = 2016\cdot 3{,}30449.. = 6661{,}8529..$

$z = 10^{6661{,}8529}$

$= 10^{6661}\cdot 10^{0{,}8529..}$

$= 7{,}12695\cdot 10^{6661}$

6661 Ziffe

erste 3: 712

e) $A = \begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\cdot a$

$A = A^3$ $\quad A^2: \begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} = \begin{pmatrix}2 & -2\\ -2 & 2\end{pmatrix}$

$A^3: 2\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} = \begin{pmatrix}4 & -4\\ -4 & 4\end{pmatrix}$

$4a^3\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} = a\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$

$4a^3 = a$

$a(4a^2-1) = 0$

$a = 0 \quad a = \pm\frac{1}{2}$