Matur ENIF18

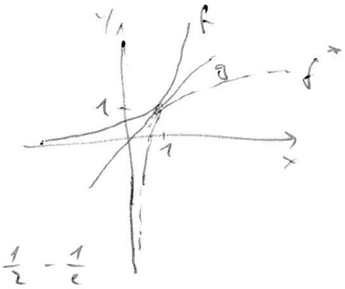
1. $f(x) = a e^x \qquad g(x) = x$

a) $a = 1 \qquad \underline{V} = \pi \int_0^1 f^2(x)\,dx = \pi \int_0^1 e^{2x}\,dx = \pi \left[\frac{1}{2} e^{2x}\right]_0^1 = \underline{\frac{\pi}{2}(e^2-1)}$

b) $f' = g' \qquad f = g$

$a e^x = 1 \to a e^x = x$

$\underline{1 = x} \longrightarrow \underline{a = \frac{1}{e}} \qquad \underline{f(x) = \frac{1}{e} e^x}$



c) $A = \int_0^1 (f-g)\,dx = \int_0^1 \left(\frac{1}{e} e^x - x\right) dx = \left[\frac{1}{e} e^x - \frac{x^2}{2}\right]_0^1$

$= 1 - \frac{1}{2} - \left(\frac{1}{e} - 0\right) = \underline{\underline{\frac{1}{2} - \frac{1}{e}}}$

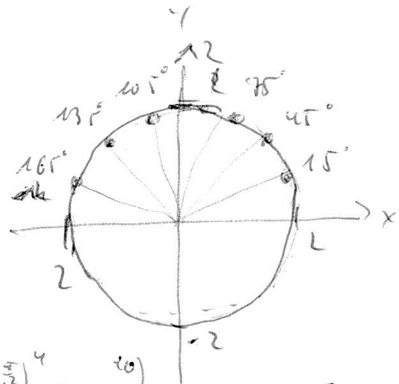
d) $y = \frac{1}{e} e^x$

$x = \frac{1}{e} e^y$

$ex = e^y$

f^* $\underline{y} = \ln(ex) = \ln e + \ln x = \underline{1 + \ln x}$

2. $z^6 = 64i = 64 e^{i\frac{\pi}{2}}$



a) $z_0 = 2 e^{i\frac{\pi}{12}} = 2\cdot\left(\cos\left(\frac{\pi}{12}\right) + i \sin\left(\frac{\pi}{12}\right)\right)$

b) $z_n = 2 \cdot e^{i\frac{\pi}{12}(2n+1)} \qquad n = 0; 1 \dots 5$

c) $z_0 + z_1 + z_2 + z_3 + z_4 + z_5$

$= 2e^{i\frac{\pi}{12}} + 2e^{i\frac{\pi}{12}\cdot 3} + \dots + 2e^{i\frac{\pi}{12}\cdot 11} = 2e^{i\frac{\pi}{12}}\left(1 + (\underbrace{e^{i\frac{\pi}{12}}}_{q})^2 + (e^{i\frac{\pi}{12}})^4 + \dots {}^{10}\right)$

$= 2e^{i\frac{\pi}{12}}(1 + q^2 + q^4 + q^6 + q^8 + q^{10}) =$

$= 2e^{i\frac{\pi}{12}}\left(1 + (q^2)^1 + (q^2)^2 + (q^2)^3 + (q^2)^4 + (q^2)^5\right) = 2e^{i\frac{\pi}{12}} \frac{1-(q^2)^6}{1-q^2}$

$q^2 = (e^{i\frac{\pi}{12}})^2 = e^{i\frac{\pi}{6}} =$

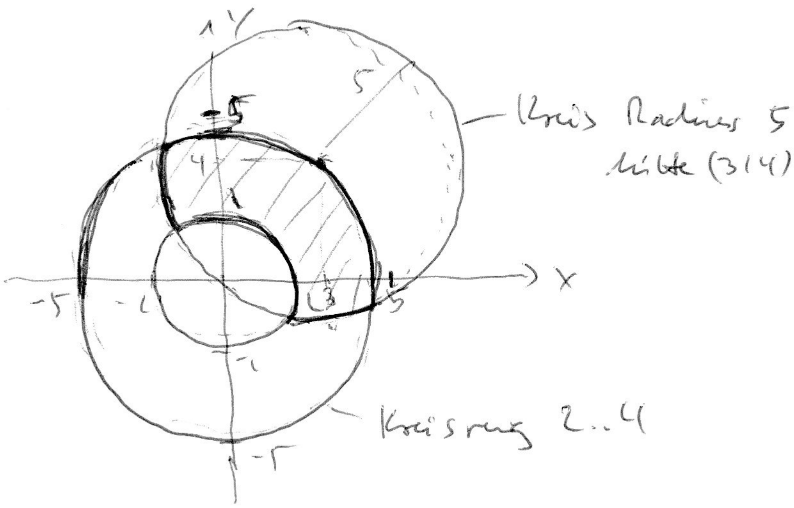
Realteile heben sich weg.;

$\underline{\Sigma} = +2(2\sin 15° + 2\sin 45° + 2\sin 75°)\,i$

$= +2\left((1+\sqrt{3})\sqrt{2}\right) i = \underline{(+2\sqrt{2} + 2\sqrt{6})\,i}$

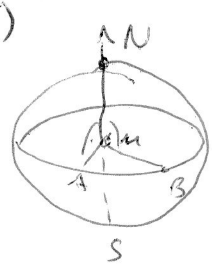
d) $\underline{\Pi} = 64 \cdot e^{i\frac{\pi}{12}(1+3+5+7+9+11)} = 64\, e^{i\pi 3} = \underline{-64i}$

e)



Matur ENF18

3. $M(1|3|4)$ $A(5|-1|-3)$ $B(2|11|0)$



a) $\vec{n} = \overline{MA} \times \overline{MB} = \begin{pmatrix} 72 \\ 9 \\ 36 \end{pmatrix} \sim \begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix}$

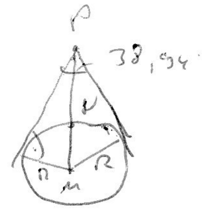
$n: \vec{x} = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} + s \underbrace{\begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix}}_{9=R} \cap K \qquad \underline{R = MA = 9}$

$N: s = +1 \quad \vec{x}_N = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \\ 8 \end{pmatrix} \qquad N(9|4|8)$

$S: s = -1 \quad \vec{x}_S = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -7 \\ 2 \\ 0 \end{pmatrix}$

b) $\vec{n} = \begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix}: \quad E: 8x + y + 4z + d = 0$

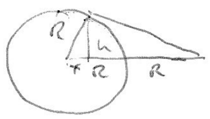
$N(9|4|8): d = -108 \qquad E: 8x + y + 4z - 108 = 0$



c) $\vec{v} = \overline{MA} \times \vec{n} \sim \begin{pmatrix} 1 \\ 8 \\ -4 \end{pmatrix} \quad t: \vec{x} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 8 \\ -4 \end{pmatrix}$

d) $MP = \frac{R}{\sin\frac{\alpha}{2}} = 27 \qquad \vec{r}_P = \vec{r}_M + 3\begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 25 \\ 6 \\ 16 \end{pmatrix} \qquad P(25|6|16)$

e)



$\frac{O_{Haube}}{O_{Kugel}} = \frac{2\pi R h}{4\pi R^2} = \frac{h}{2R} = \underline{\frac{1}{4} = 25\%} \leftarrow \quad x \cdot 2R = R^2, \; x = \frac{R}{2} \Rightarrow h = \frac{R}{2}$

4. a) $2^{18} = 262144$

b) $\binom{18}{10} = \frac{18!}{8!\,10!} = 43758$

c) $P(\text{mind. ein Spalt schwarz}) = 1 - P(\text{kein Spalt schwarz}) = 1 - \left(\frac{7}{8}\right)^6 = 55{,}1\%$ $\qquad P(\blacksquare) = \frac{1}{8}$, $P(\square) = \frac{7}{8}$

d) $P(5 \text{ von } 12 \text{ Körner schwarz}) = \binom{12}{5}\left(\frac{8}{18}\right)^5\left(\frac{10}{18}\right)^7 = 22{,}4\%$

e) $P(sw) + P(ws) = \frac{x}{18} \cdot \frac{18-x}{18} \cdot 2 = \frac{5}{18}$ $\qquad x$ ~~weiße~~ schwarz

$x = 3$

$x = 15$

5a) $\lg\left(\frac{21}{4}y\right) = \lg\left(\frac{21}{4} + y\right) \Rightarrow \frac{21}{4}y = \frac{21}{4} + y \Rightarrow y = \frac{21}{17}$

b) $\left|\frac{1 + x - \frac{1}{1-x}}{\frac{1}{1-x}}\right| < 0{,}1 \Rightarrow |-x^2| < 0{,}1 \Rightarrow \underline{|x| < \sqrt{0{,}1}} \qquad x \in \left]-\frac{1}{\sqrt{10}} ; \frac{1}{\sqrt{10}}\right[$

c) $\left.\begin{array}{l} (v^2 - u^2)' = 2vv' - 2uu' = 2vu - 2uv = 0 \\ g(0) = v^2(0) - u^2(0) = 1 - 1 = 0 \end{array}\right\} \; g(x) \equiv 0 \qquad g$ ist identisch 0

d) $a = 4, \; b = 3 \quad S(2|4) \qquad 3^2(x-2)^2 + 4^2(y-4)^2 = (3 \cdot 4)^2$

e) $n = 1: \quad \frac{1}{2!} = 1 - \frac{1}{2!}$ ✓ Induktionsanfang

$n+1: \quad \underbrace{\frac{1}{1!} + \frac{2}{3!} + \dots + \frac{n}{(n+1)!}}_{} + \frac{n+1}{(n+2)!} \stackrel{!}{=} 1 - \frac{1}{(n+2)!}$

$1 - \frac{1}{(n+1)!} + \frac{n+1}{(n+2)!}$

$1 - \frac{1}{(n+1)!} + \frac{n+1}{(n+2)(n+1)!}$

$\frac{(n+1)!(n+2) - (n+2) + n+1}{(n+2)(n+1)!} = \frac{(n+1)!(n+2) - 1}{(n+2)(n+1)!} = \frac{(n+2)! - 1}{(n+2)!} = 1 - \frac{1}{(n+2)!}$ $\Bigg] \Rightarrow$ ✓ Induktionsschluss