

1.  $f(x) = a - x^2 ; a > 0$

a)  $x = \pm \sqrt{a} \quad a = n^2 \rightarrow x = \pm n \quad n \in \mathbb{N}_n$

b)  $a = 27 \quad x = \pm 3\sqrt{3}$



$A = x \cdot (a - x^2) ; x \in [0; 3\sqrt{3}]$

$= ax - x^3$

$A' = a - 3x^2 \rightarrow A' = 0 \quad x = \pm \sqrt{\frac{a}{3}} = \pm \sqrt{\frac{27}{3}} = \pm 3$

$A'' = -6x \quad A''(3) < 0 \Rightarrow \text{Max}$

$A_{\text{max}} = 54$

$A(0) = 0$   
 $A(3\sqrt{3}) = 0$  } Min

c)  $f'(\sqrt{a}) = -\tan 60^\circ$

$-2\sqrt{a} = -\sqrt{3}$

$a = \frac{3}{4}$

$f'(x) = -2x$

d)  $a = 1 : \left. \begin{matrix} f_1(x) = 1 - x^2 \\ g(x) = \cos(bx) \end{matrix} \right\} \text{NST}$

$x = 1$   
 $b = \frac{\pi}{2}$   
 $h = \frac{\pi}{2}$

$A_f = \int_0^1 f_1(x) dx = [x - \frac{1}{3}x^3]_0^1 = \frac{2}{3}$   
 $A_g = \int_0^1 g(x) dx = [\frac{2}{\pi} \sin(\frac{\pi}{2}x)]_0^1 = \frac{2}{\pi}$

e)  $f_4(x) = 4 - x^2 \quad \text{NST} : x = \pm 2$

$V = \pi \int_{-2}^2 (4 - x^2) dx = 2\pi [4x - \frac{8}{3}x^3 + \frac{1}{5}x^5]_0^2 = \frac{512}{15}\pi$

A. Sym.  
alle Exp. grade

2.  $z_0 = 1 \quad z_k = z_{k-1} \cdot \left( \frac{1}{L} \sqrt{3} + \frac{1}{L} i \right) \quad k \in \mathbb{N}$

a)  $|q| = 1 \quad \varphi = \arctan\left(\frac{1/L}{1/L\sqrt{3}}\right) = \frac{\pi}{6}$

$q = e^{i\frac{\pi}{6}} = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$

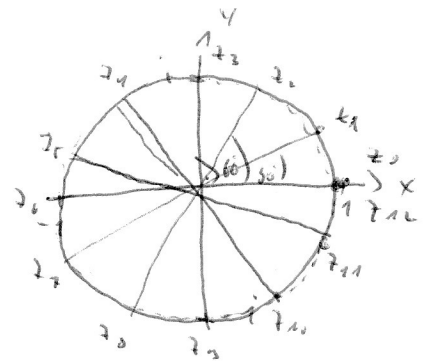
$z_k = z_{k-1} \cdot q = z_{k-2} \cdot q^2 = \dots = q^{k \cdot a} \cdot z_0$

$z_0 = 1 \quad k \in \mathbb{N}_0$

$= \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)^k$

b)

c)  $A = 12 \cdot \frac{1}{L} \cdot 1 \cdot 1 \cdot \sin 30^\circ = 6 \cdot \frac{1}{L} = 3$



$U = 12 \cdot 2 \cdot 1 \cdot \sin 15^\circ = 6,2117$

d)  $w = a + ib \quad v = c + id$

$\bar{w} \cdot \bar{v} = (a - ib)(c - id) = ac - bd - i(ad + bc)$

$\overline{wv} = \overline{(a + ib)(c + id)} = \overline{ac + bd + i(ad + bc)} = ac + bd - i(ad + bc)$  } = v

e)  $\left| \frac{w}{v} \right|^2 = \left| \frac{a + ib}{c + id} \right|^2 = \frac{|(a + ib)(c - id)|^2}{|c + id|^2} = \frac{|ac + bd + i(-ad + bc)|^2}{c^2 + d^2}$

f)  $\frac{|w|^2}{|v|^2} = \frac{a^2 + b^2}{c^2 + d^2} = \frac{(ac + bd)^2 + (bc - ad)^2}{(c^2 + d^2)^2} = \frac{a^2c^2 + 2abcd + b^2d^2 + b^2c^2 - 2abcd + a^2d^2}{(c^2 + d^2)^2} = \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2} = \frac{a^2 + b^2}{c^2 + d^2}$

3. a)  $g \vec{w} \vec{E} : 2(-2+4) + 5(-1+7) + 1 \cdot ((1+1)+) = 450$   
 $+ = 2$

b)  $w_{\text{rot}} = \frac{\begin{pmatrix} 4 \\ 7 \\ 13 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\sqrt{234} \sqrt{225}} = \frac{215}{15}$   $D(6|12|27)$   
 $w = 14,36$

c)  $d = \left| \frac{2x + 5y + 1 \cdot (-450)}{15} \right|_{(0,0)} = 30$

d)  $M(215|14) \quad R = 33$   
 $d_M = \left| \frac{2 \cdot 2 + 5 \cdot 5 + 14 \cdot 44 - 450}{15} \right| = 15$   $b = \sqrt{R^2 - d^2} = 36$

e)  $V: \vec{X} = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 4 \\ 7 \\ 13 \end{pmatrix} + s \begin{pmatrix} 6 \\ 6 \\ 1 \end{pmatrix}$   $x=0 \Rightarrow z=0$   
 $\Rightarrow 1+13t+s=0$   
 $s = -1-13t$   
 $\vec{X} = \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 4 \\ 7 \\ 6 \end{pmatrix}$   
 $y = \frac{7}{4}(x+2) - 2 = \frac{7}{4}x + \frac{13}{2}$

4 a)  $P(3 \times 5' \text{ in } 6) = \binom{6}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^3 = 5,36\%$

b)  $P(\text{jeder Wurf}) = 6! \left(\frac{1}{6}\right)^6 = 1,54\%$

c)  $P(123456) = \left(\frac{1}{6}\right)^6 = 0,00014\%$

d)  $P(6\bar{6}) = \frac{2}{6} = 2P(1-p) \quad p = 1/6 \text{ oder } p = 5/6$

e)  $7 = 1+6+2+5 = 3+4 \Rightarrow 6 \times 36 \quad p = \frac{1}{6} \quad E = n \cdot p = 30$   
 $\sigma = \sqrt{n \cdot p \cdot q} = 5$

5 a)  $c^2 = a^2 + b^2 - 2ab \cos 30^\circ$

$\sqrt{3} = \frac{a^2 + b^2 - c^2}{2ab}$   $a, b, c$  ganze Zahlen  $\rightarrow a^2, b^2, c^2$  auch, ab ebenso, also  $\frac{a^2 + b^2 - c^2}{2ab}$  Bruch  $\neq \sqrt{3}$

b)  $B_A = 12 B_B \quad B_A \cdot 0,92^n = B_B \cdot 1,05^n$   
 $12 = \left(\frac{1,05}{0,92}\right)^n \rightarrow n = \log_{\frac{1,05}{0,92}} 12 = 18,8$  nach 19 Jahren

c)  $x^3 + 3x^2 - 134x + 648 = 0 \quad x_1 = 3$   
 $:(x-3) = x^2 + 6x - 216$   
 $(x+18)(x-12)$   
 $x = -18; x = 12$

d)  $n = 4^{(5^6)} = 4^{15625}$   
 $\lg n = 15625 \cdot \lg 4 = 15625 \cdot 0,602 = 9467,18$   
 $n = 10^{9467,18} = 10^{9467} \cdot 10^{0,18} = 1,5394 \cdot 10^{9467}$   
 $\frac{153}{1000} \text{ unter 3} \quad 9467 \text{ Stellen}$

e)  $\vec{7A} \cdot \vec{2D} = 0$   
 $\begin{pmatrix} 16 \\ 6 \\ 10 \end{pmatrix} \cdot \begin{pmatrix} -8 \\ 2 \\ 2 \end{pmatrix} = 0$   
 $-128 + (20-2)(2-2) = 0$   
 $z = -6$   
 $t = 18$