LEN|F2L

1. $f(x) = \frac{x^3 - a}{2x^2}$ $a > 0$

a) $f(x) = \frac{1}{2}x - \frac{a}{2}x^{-2}$ $D = \mathbb{R}\setminus\{0\}$

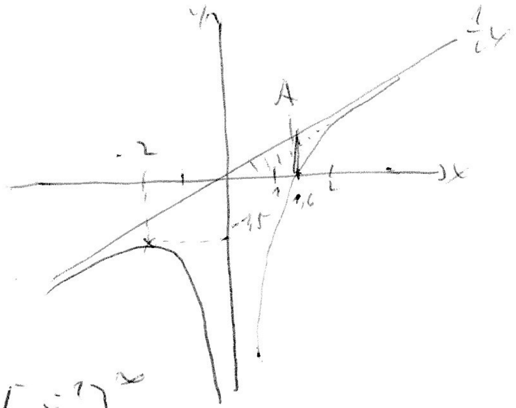
$f'(x) = \frac{1}{2} + ax^{-3}$ NST: $\underline{x = \sqrt[3]{a}}$

$f''(x) = -3ax^{-4}$ Ext: $\underline{x = -\sqrt[3]{2a}}$ $y = -\frac{3}{4}\sqrt[3]{2a}$ $f''<0 \Rightarrow \underline{Max(\sqrt[3]{2a} \mid -\frac{3}{4}\sqrt[3]{2a})}$

WP: $\frac{-3a}{x^4} = 0$ (f) kein WP

b) $D = \mathbb{R}\setminus\{0\}$ $W = \mathbb{R}$

$y = \frac{1}{2}x$ (s.o.)



c) $A = A_\Delta + A_{\triangle}$

$= \frac{1}{2}\sqrt[3]{a}\cdot\frac{1}{2}\sqrt[3]{a} + \int_{\sqrt[3]{a}}^{\infty}\left(\frac{1}{2}x - f(x)\right)dx$

$= \frac{1}{4}\sqrt[3]{a^2} + \int_{\sqrt[3]{a}}^{\infty}\left(+\frac{a}{2}x^{-2}\right)dx = \frac{1}{4}\sqrt[3]{a^2} + \frac{a}{2}\left[-x^{-1}\right]_{\sqrt[3]{a}}^{\infty}$

$= \frac{1}{4}\sqrt[3]{a^2} + \frac{a}{2}\frac{1}{\sqrt[3]{a}} = \underline{\frac{3}{4}\sqrt[3]{a^2}}$

d) $a = 4$ $P(0 \mid -1{,}5)$: y_{max} : t_1: $\underline{y = -1{,}5}$

$$\frac{\frac{x^3-4}{2x^2} + 1{,}5}{x - 0} = \frac{1}{2} + 4x^{-3}$$

$$\frac{x^3-4}{2x^2} + \frac{3}{2} = \frac{1}{2}x + 4x^{-2}$$

$$x^3 - 4 + x^2 = x^3 + 8$$

$$x^2 = 4$$

$\underline{x = \pm 2}$ $y_1 = -1{,}5$ $T_1(-2 \mid -1{,}5)$ $\underline{t_1: y = -1{,}5}$

$y_2 = 1{,}2$ $T_2(2 \mid 1{,}2)$ $\underline{t_2: y = x - 1{,}5}$

e) $\underline{\tilde{f}} = f(x-2) + \frac{3}{2} = \underline{\frac{x^3 - 3x^2}{2(x-2)^2}}$

$\underline{\tilde{a}} = a(x-2) + \frac{3}{2} = \underline{\frac{1}{2}x + \frac{1}{2}}$

EN | Ku

2 a) $\vec{AB} = \begin{pmatrix}6\\12\\0\end{pmatrix}$ $\vec{BC} = \begin{pmatrix}-8\\4\\10\end{pmatrix}$ $\vec{CD} = \begin{pmatrix}-6\\-12\\0\end{pmatrix}$ $\vec{DA} = \begin{pmatrix}+8\\-4\\-10\end{pmatrix}$

$\underbrace{\vec{AB} = -\vec{CD} \quad \vec{BC} = -\vec{DA}}_{\text{ebenes Parallelogramm}} \longrightarrow \left.\begin{array}{l} AB = \sqrt{180} \\ BC = \sqrt{180} \end{array}\right\}$ Raute $\longrightarrow \vec{AB} \cdot \vec{BC} = -48 + 48 + 0 = 0$

$\Box$ Quadrat

b) $G = 180$ $\quad V = \frac{1}{3} G h$

$h = 3V/G = \frac{3 \cdot 360}{180} = 6$

Mitte ABCD: $\vec{r}_M = \frac{\vec{r}_A + \vec{r}_C}{2} = \begin{pmatrix}3\\8\\7\end{pmatrix}$

$\vec{AB} \times \vec{BC} = \begin{pmatrix}120\\-60\\120\end{pmatrix} \sim \begin{pmatrix}2\\-1\\2\end{pmatrix} = \vec{n}$

$|\vec{n}| = 3$

$\vec{MS} = \pm 2\vec{n} = \pm\begin{pmatrix}4\\-2\\4\end{pmatrix}$

$\vec{s}_1 = \vec{r}_M + \vec{MS} = \begin{pmatrix}7\\6\\11\end{pmatrix}$ $S_1(7|6|11)$

$\vec{s}_2 = \vec{r}_M - \vec{MS} = \begin{pmatrix}-1\\10\\3\end{pmatrix}$ $S_2(-1|10|3)$

c) $O = G + 4 \cdot A_{ABS}$

$A_{ABS} = \frac{1}{2}|AB \times AS| = \frac{1}{2}\left|\begin{pmatrix}6\\12\\0\end{pmatrix} \times \begin{pmatrix}3\\6\\9\end{pmatrix}\right| = \frac{1}{2}\left|\begin{pmatrix}108\\-54\\0\end{pmatrix}\right| = 27\sqrt{5}$

$O = 180 + 4 \cdot 27\sqrt{5} = \underline{180 + 108\sqrt{5}}$

d) $\vec{n}_{ABS} = \begin{pmatrix}108\\-54\\0\end{pmatrix} \sim \begin{pmatrix}2\\-1\\0\end{pmatrix}$: $2x - y + d = 0$

$A(4|0|1)$ $\quad 8 + d = 0$

$E: \underline{2x - y - 8 = 0}$

$\cos\alpha = \frac{\vec{n} \cdot \vec{n}_{ABS}}{n \cdot n_{ABS}} = \frac{\begin{pmatrix}2\\-1\\2\end{pmatrix}\begin{pmatrix}2\\-1\\0\end{pmatrix}}{3\sqrt{5}} = \frac{5}{3\sqrt{5}} = \frac{\sqrt{5}}{3} \Rightarrow \underline{\alpha = 41{,}81°}$

e) R: Abstand M zu Seitenfläche: $\frac{2x - y - 8}{\sqrt{5}} = 0$

$M(3|8|7)$: $\left|\frac{6 - 8 - 8}{\sqrt{5}}\right| = \underline{2\sqrt{5} = R}$

K: $(x-3)^2 + (y-8)^2 + (z-7)^2 = 20$

EM|F12

3, a) $a_1)$ $13!$

$a_2)$ $4! \cdot 9!$

$a_3)$ $4 \cdot 4! \cdot 9!$

b) $P(10\ \text{Punkte}) = \frac{4}{13}$

c) P(16 Punkte) : $16 = 11+5$ (2x) =

$= 10+6$ (2x)

$= 9+7$ (2x)

$= \cancel{8+8}$

$P(16\ \text{Punkte}) = 2\cdot\left(\frac{1}{13}\right)^2 + 2\cdot\frac{4}{13}\cdot\frac{1}{13} + 2\cdot\left(\frac{1}{13}\right)^2 = \frac{12}{169} = 7{,}1\,\%$

d) P(höchstens 5 P) = $2+3$ (2x) $3 \cdot \frac{1}{13^2}$

$2+2$ ~~(2·1x)~~

$P(\text{in } N \text{ Versuchen mind. einmal nur } 5P) \geq 95\,\%$

$1 - P(\text{" keinmal .. "}) \geq 0{,}95$

$\left(\frac{166}{169}\right)^N \leq 0{,}05$

$N \geq \log_{\frac{166}{169}} 0{,}05 = 167{,}257$

Ab 168 Versuche

$P(11P) = \frac{1}{6}\cdot\frac{1}{13}\cdot 4 = \frac{2}{39} = 5{,}13\,\%$

e) $P(12P)$: $12 = 2\cdot 6$

$3\cdot 4$

$4\cdot 3$

$6\cdot 2$

f) $E(1\times w, 1\times z) = \underbrace{\frac{1}{6}\cdot 1}_{w}\cdot\underbrace{\frac{1}{13}\cdot 2}_{k} + \underbrace{\frac{1}{6}\cdot 1}_{w}\cdot\underbrace{\frac{1}{13}\cdot 3}_{k} + \ldots + \frac{1}{6}\cdot\underbrace{1}_{w}\cdot\frac{1}{13}\cdot\underbrace{11}_{k}$

$+ \frac{1}{6}\cdot 2 + \frac{1}{13}\cdot 2 + \frac{1}{6}\cdot 2\cdot\frac{1}{13}\cdot 3 + \ldots$

$\vdots$

$= \frac{1}{6}\cdot\frac{1}{13}\cdot 1\,(\underbrace{2+3+4+\ldots 10+10+10+10+11}_{k})$

$+ \frac{1}{6}\cdot\frac{1}{13}\cdot 2\cdot(\quad " \quad)$

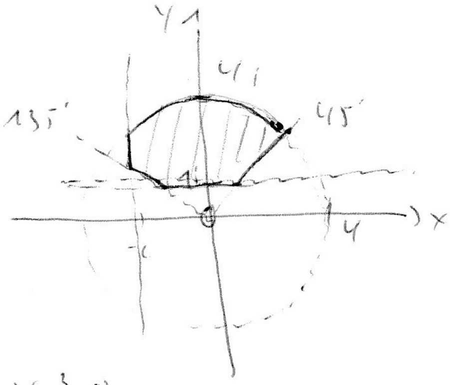
$+ \frac{1}{6}\cdot\frac{1}{13}\cdot 3\,(\quad " \quad)$

$= \frac{1}{6}\cdot\frac{1}{13}\cdot(\underbrace{1+2+3+4+5+6}_{w})\,k$

$= \frac{1}{6}\cdot\frac{1}{13}\,k\cdot w = \frac{1}{6}\cdot\frac{1}{13}\cdot 21\cdot 95$

$E = 25{,}6$

4.1 a) (ENIF...)



b) $\underbrace{(z^2-i)}\underbrace{(z^3+8)} = 0$

$\underline{z = \pm i}$ $\quad z = 2 \quad (0°)$

$\underline{z = -1 \pm \sqrt{3}i} \quad (120° | 240°)$

c) $z = 1-i$ $\quad z^n = (\bar{z})^n$

$-45°$

Potenzieren : $n \cdot 4$

$\bar{z} = 1+i$

$45°$

0°	±45°	±90°	±135°	±180°	±225°	±270°	±315°	±360°
0	1	2	3	4				8

Für alle Vielfachen von 4 (inkl. 0, falls 0 in $\mathbb{N}$)

4.2. $A = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$

a) $A\bar{v} = \lambda\bar{v}$

$(A - \lambda \mathbb{1})\bar{v} = 0$

$\det\begin{pmatrix} 4-\lambda & 3 \\ 1 & 2-\lambda \end{pmatrix} = 0$

$(4-\lambda)(2-\lambda) - 3 = 0$

$\lambda^2 - 6\lambda + 5 = 0$

$\underline{\lambda_1 = 5}$

$\underline{\lambda_2 = 1}$

$\lambda_1 = 5 \quad \begin{pmatrix} -1 & 3 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-a + 3b = 0$

$\underline{\bar{v}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}}$

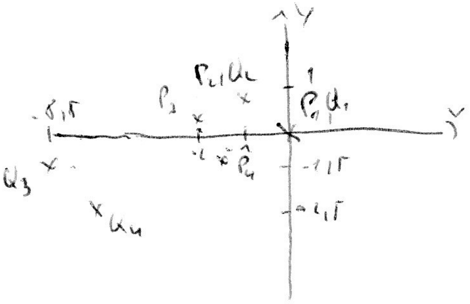
$\lambda_2 = 1 \quad \begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$a + b = 0$

$\underline{\bar{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$

b) $Q_1 = A \cdot P_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad Q_3 = A \cdot P_3 = \begin{pmatrix} -8{,}5 \\ -1{,}5 \end{pmatrix}$

$Q_2 = A \cdot P_2 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ $\quad Q_4 = A \cdot P_4 = \begin{pmatrix} -7{,}5 \\ -2{,}5 \end{pmatrix}$



c) $\underline{\det(A) = 4 \cdot 2 - 3 \cdot 1 = \underline{5}}$

Also 5-fache Fläche

d) $\underline{y_1 = \frac{1}{3}x}$

$\underline{y_2 = -x}$

Ein Punkt auf y_1, z.B. P(6|2) bleibt auf der Geraden, aber seine Koordinaten werden ver-5-facht Q(30|10) (Eigenwert 5)

Auch ein Punkt auf dieser Geraden bleibt auf ihr, aber wegen Eigenwert 1 ändern sich seine Koordinaten nicht, z.B. P(-8|8) → Q(-8|8)

(EMF22)

5.1. a) $\overline{AC} = \cos\alpha \quad \overline{CB} = \sin\alpha$

$AP = AC \cdot \cos\alpha = \cos^2\alpha$

$CP = AC \cdot \sin\alpha = \cos\alpha \cdot \sin\alpha$ $\Big\}\ A_{APC} = \frac{1}{2} AP \cdot CP = \underline{\frac{1}{2} \sin\alpha \cos^3\alpha}$

b) $A' = \frac{1}{2}\left(\cos^4\alpha - 3\sin^2\alpha\cos^2\alpha\right) = 0 \qquad D = [0;90]$

$\underbrace{\cos^2\alpha}\left(\cos^2\alpha - 3\sin^2\alpha\right) = 0$

$\underline{\alpha = 90°} \qquad \tan\alpha = \pm\frac{1}{\sqrt{3}}$

$\underline{\alpha = \pm 30°}$

Vorzeichentabelle (stetige Funktion)

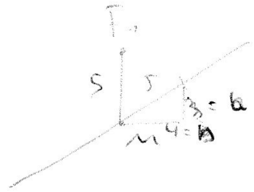
α	0°	30°	45°
$A'(\alpha)$	+	0	−

$\nearrow \ \xrightarrow{} \ \searrow$ Max

$\underline{\alpha = 30°}$

Ränder D: $A(0°) = 0$, $A(90°) = 0$ $\Big\}$ Min

5.2. a) $F_1(4|3) \quad F_2(4|-7)$



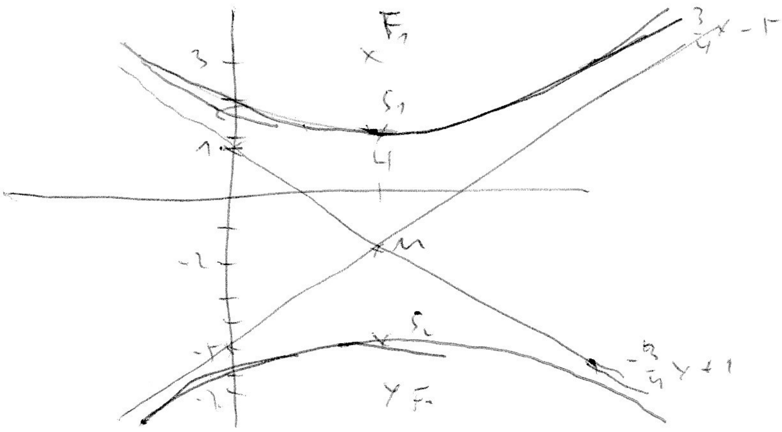
$a_1: \ y = \frac{3}{4}x - 5$

$M(4|-2) = \frac{\vec{r}_{F_1} + \vec{r}_{F_2}}{2}$

$a = 3: \quad S_1(4|1)$

$S_2(4|-5)$

e: $9(x-4)^2 - 16(y+2)^2 + 144 = 0$



b) $\hat{a}_1: \ y = -\frac{4}{3}(x-4) - 2$

$\hat{a}_2: \ y = \frac{4}{3}(x-4) - 1$

5.3 a) $2e^{2x} - e^{3-x} = 0$

$2e^{3x} - e^{3} = 0$

$e^{3x} = \frac{1}{2}e^3$

$3x = \ln\left(\frac{1}{2}e^3\right)$

$\underline{x = \frac{1}{3}\ln\left(\frac{1}{2}e^3\right)}$

b) $2\ln(x+1) = \ln(2x+6) - \ln 2 \qquad D =]-1;\infty[$

$\ln(x+1)^2 = \ln(x+3)$

$\frac{(x+1)^2}{x+3} = 1$

$\underline{x = 1}$

$(x = -2) \notin D$