EW; HW

1. $f_a(x) = a^2 x e^{-\frac{x}{a}}$; $a>0$

a) $f_a'(x) = (a^2 - ax)\, e^{-\frac{x}{a}}$

$f_a''(x) = (x-2a)\, e^{-\frac{x}{a}}$

$f_a'(x) = 0$

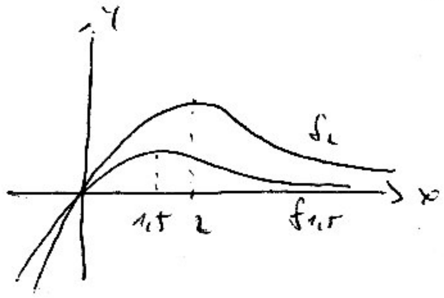
$\underline{x = a}$ $\underline{y = \frac{a^3}{e}}$

$f''(a) = -a e^{-1} < 0 \Rightarrow \underline{Max(a \mid \frac{a^3}{e})}$

$a = 2$ $\underline{Max(2 \mid \frac{8}{e})}$

b) $f_{1,5}(x) = \frac{9}{4} x e^{-\frac{2}{3}x}$

$f_2(x) = 4x e^{-\frac{x}{2}}$



c) $f'(0) = \tan 60° = \sqrt{3}$

$a^2 = \sqrt{3}$

$\underline{a = \underset{(-)}{+} \sqrt[4]{3}}$

d) WP: $f_a''(x) = 0$

$x = 2a$; $y = 2a^3 e^{-2}$

$a = \frac{x}{2} \rightarrow \underline{y} = 2\frac{x^3}{2^3} e^{-2} = \underline{\frac{1}{4} e^{-2} x^3}$

e) $F_a(x) = -a^3 (a+x)\, e^{-\frac{x}{a}}$

$\underline{F_a'(x)} = -a^3 \left(1 \cdot e^{-\frac{x}{a}} + (a+x) e^{-\frac{x}{a}} \cdot \left(-\frac{1}{a}\right)\right)$

$= a^2 x e^{-\frac{x}{a}} = \underline{f(x)}$

$A = 256 = \lim_{b \to \infty} \int_0^b f(x)\, dx = \lim_{b \to \infty} \left[-a^3 (a+x) e^{-\frac{x}{a}} \right]_0^b$

$256 = \lim_{b \to \infty} \underbrace{-a^3 \underbrace{(a+b)}_{\to \infty} \underbrace{e^{-\frac{b}{a}}}_{\to 0}}_{\to 0} + a^4$ Exp. stärker als jede Potenz

$256 = a^4$

$\underline{a = 4}$

EW, H10

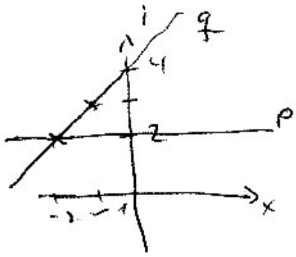
2. a) $z^3 = 8i$

$z^3 = 8 \cdot e^{i(\frac{\pi}{2} + k \cdot 2\pi)}$

$z = 2 \cdot e^{i(\frac{\pi}{6} + \frac{k}{3} 2\pi)}$ $\quad k = 0, 1, 2$

b) $f: w = f(z) = (1+i)z$

$p: \operatorname{Im}(z) = 2 \xrightarrow{f} q$?



2 Punkte auf p: $z_1 = 2i$ $\quad w_1 = (1+i)z_1 = -2+2i$

$z_2 = 2i + 1$ $\quad w_2 = (1+i)z_2 = -1+3i$

$\underline{q: \operatorname{Im}(w) = 1 \cdot \operatorname{Re}(w) + 4}$

$\underline{u = 1 \; ; \; v = 4}$

c) $f(z) = (12-5i)z + 5 = w$

$z \leftrightarrow w$: $z = (12-5i)w + 5$

$w = \frac{z-5}{12-5i} = \frac{(z-5)(12+5i)}{12^2 - (5i)^2} = \frac{(z-5)12 + 5(z-5)i}{169}$

$w = \frac{12}{169}(z-5) + \frac{5}{169}(z-5)i$

$\bar{f}: w = \frac{1}{169}(12+5i)z + - \frac{5}{169}(12+5i)$

d) $h(x)$: $\underbrace{\text{Drehung um } 30°\text{, Faktor } 2}$ um $z_0 = 4+3i$ $\quad h(z) = az + b$

$a = 2 \cdot e^{i\frac{\pi}{6}} = 2 \cdot (\cos\frac{\pi}{6} + i \sin\frac{\pi}{6})$

$\underline{a} = 2 \cdot (\frac{1}{2}\sqrt{3} + i \sin\frac{1}{2}) = \underline{\sqrt{3} + i}$

$h(z_0) = z_0 = az_0 + b$

$b = (1-a)z_0 = (\sqrt{3}+i)(4+3i) = 4\sqrt{3} - 3 + (3\sqrt{3}+4)i$

$\underline{h(z) = (\sqrt{3}+i)z + 4\sqrt{3} - 3 + (3\sqrt{3}+4)i}$

e) $z = 1 + \cos\alpha + i \sin\alpha$;

$|z| \leq 1$?

$z = 1 + e^{i\alpha}$

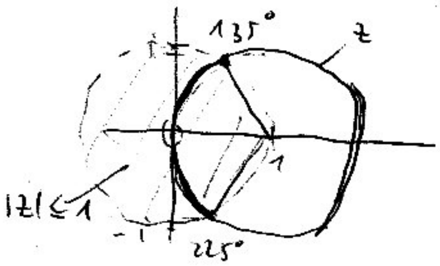
Kreis Radius 1

$|z|^2 = (1 + e^{i\alpha})(1 + e^{-i\alpha})$

$|z|^2 = 2 + 2\cos\alpha \leq 1$

$\cos\alpha \leq \frac{1}{2}$

$\underline{135° \leq \alpha \leq 225°}$



$\underline{\alpha \in [135°; 225°]}$

EW; H10

3. $E: x-2y-2z=0 \quad F: x+y-z=0 \quad M(11|13|12)$

Lot von M auf $E \to B$

a) $HNF(E) = \frac{x-2y-2z}{3} = 0$

$M(11|13|12)$; $\left|\frac{11-2\cdot 13-2\cdot 12}{3}\right| = d(M;E)$

$$\underline{13 = d(M;E)}$$

b) B? $\quad g_B: \vec{x} = \begin{pmatrix}11\\13\\12\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\-2\end{pmatrix} \cap E:$

$11+\lambda-2(13-2\lambda)-2(12-2\lambda)=0$

$\lambda = \frac{13}{3}$

$\vec{r}_0 = \begin{pmatrix}11\\13\\12\end{pmatrix} + \frac{13}{3}\begin{pmatrix}1\\-2\\-2\end{pmatrix} = \frac{1}{3}\begin{pmatrix}46\\13\\10\end{pmatrix} \qquad \underline{B\left(\frac{46}{3}\middle|\frac{13}{3}\middle|\frac{10}{3}\right)}$

c) $E \cap F$: $\quad F: x = z-y$

in E: $z-y-2y-2z=0$

$z=-3y$ in F: $x=-4y$

$\underline{s: \vec{x} = \begin{pmatrix}x\\y\\z\end{pmatrix}} = \begin{pmatrix}-4y\\y\\-3y\end{pmatrix} = \underline{y\cdot\begin{pmatrix}-4\\1\\-3\end{pmatrix}}$

d) $C(-1|9|9)$; $g \parallel F$; $C \in g$; $g \perp (MC)$

Richtungsvektor $\vec{v}$ von g : $\left.\begin{matrix}\vec{v} \perp \vec{MC}\\ \vec{v} \perp \vec{n}_F\end{matrix}\right\}$ $\vec{v} \sim \vec{MC} \times \vec{n}_F = \begin{pmatrix}12\\4\\3\end{pmatrix} \times \begin{pmatrix}1\\1\\-1\end{pmatrix}$

$= \begin{pmatrix}-7\\15\\8\end{pmatrix}$

$g: \vec{x} = \begin{pmatrix}-1\\9\\9\end{pmatrix} + \lambda\begin{pmatrix}-7\\15\\8\end{pmatrix}$

e) $d(M;h)$ $\quad h: (0|0|0) \in h,\ P(5|8|8) \in h$

$h: \vec{x} = \lambda\underbrace{\begin{pmatrix}5\\8\\8\end{pmatrix}}_{\vec{v}}$

$\underline{\underline{d(M;h)}} = \frac{|\vec{OM} \times \vec{v}|}{v} = \frac{\left|\begin{pmatrix}11\\13\\12\end{pmatrix} \times \begin{pmatrix}5\\8\\8\end{pmatrix}\right|}{\sqrt{153}} = \frac{\left|\begin{pmatrix}8\\-28\\23\end{pmatrix}\right|}{\sqrt{153}} = \frac{9\sqrt{17}}{3\sqrt{17}} = \underline{\underline{3}}$

EW ; HÜ

$n \geq 3$

4. a) 3 blaue Seiten (Ecken): 8 $= 8$

2 blaue Seiten (Seitenkanten, ohne Ecken): $(n-2) \cdot 12$ $= 12(n-2)$

1. blaue Seite (Seitenfläche, ohne Kante): $(n-2)^2 \cdot 6 =$ $= 6(n-2)^2$

0 " " (Würfelinneres) : $(n-2)^3$ $= (n-2)^3$

b) WS (2mal Z. mit Z. gibt 2 blaue Flächen)

$= WS(0;2) \cdot 2 + WS(1;1) = 2 \cdot \frac{(n-2)^3 \cdot 12(n-2)}{n^6} + \frac{36(n-2)^4}{n^6} = \underline{60 \frac{(n-2)^4}{n^6}}$

$\underline{n=4}$

$n_3 = 8$; $n_2 = 24$; $n_1 = 24$; $n_0 = 8$

c) Erwartungswert (Anzahl blaue Flächen) bei einmaligem Ziehen

$\underline{\mu} = 3 \cdot p_3 + 2p_2 + 1p_1 + 0p_0 = 3 \cdot \frac{8}{64} + 2 \cdot \frac{24}{64} + 1 \cdot \frac{24}{64} = \underline{\frac{3}{2}}$

d) n Ziehen mit Z., damit : WS(mind. ein unbemaltes) $\geq 95\%$

$p_0 = \frac{8}{64}$

$\bar{p}_0 = \frac{56}{64}$

$1 - WS(\text{kein unbemaltes}) > 0{,}95$

$WS(\;"\;) < 0{,}05$

$\left(\frac{56}{64}\right)^n < 0{,}05$

$n > \log_{\frac{56}{64}} 0{,}05 = 22{,}43.$

$\underline{n \geq 23}$

e) 448mal Z. mit Z. 68%-Intervall

$p_0 = \frac{8}{64}$ $\hat{=}$ 1σ-Umgebung

$\sigma = \sqrt{npq} = \sqrt{448 \cdot \frac{8}{64} \cdot \frac{56}{64}} \doteq 7$

$\mu = np = 56$

$W(\mu - \sigma \leq X \leq \mu + \sigma) = 68\%$

$\underline{W(49 \leq X \leq 63) = 68\%}$

EW; H10

5. a) $\vec{u} = \begin{pmatrix} t \\ 1 \\ 2 \end{pmatrix}$; $\vec{v} = \begin{pmatrix} t \\ 2 \\ 1 \end{pmatrix}$ max Winkel

$\cos\alpha = \frac{\vec{u}\cdot\vec{v}}{u \cdot v} = \frac{t^2+4}{\sqrt{t^2+5}^2} = \frac{t^2+4}{t^2+5} > 0$ α max wenn $\cos\alpha$ min

$(\cos\alpha)' = \frac{2t}{(t^2+5)^2} = 0 \Rightarrow t = 0$ VZW $(-) \to (+)$, also Min $t = 0$

$\cos\alpha = \frac{4}{5}$

$\underline{\alpha_1 = 36{,}9° = 37°}$

$\underline{\underline{(\alpha_2 = 323°)}}$

b) $\lim\limits_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{e^{4x-\pi} - 1} = \lim\limits_{x \to \frac{\pi}{4}} \frac{\cos x + \sin x}{4e^{4x-\pi}} = \frac{\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2}}{4} = \underline{\underline{\frac{1}{4}\sqrt{2}}}$

c) $z = 9119^{2112}$

$\lg z = 2112 \lg 9119 = 8363 + 0{,}40851958$

$z = 10^{8363} \cdot 10^{0{,}4085...}$

$= 10^{8363} \cdot 2{,}5616487...$

<u>erste 5 Ziffern</u>: $\underline{\underline{25616}}$

letzte 2 Ziffern: $(9100 + 19)^n = \ldots + n \cdot 9100 \cdot 19^{n-1} + 19^n$

$\ldots \binom{n}{2} 9100^2 \cdot 19^{n-2}$

für die letzten zwei Ziffern kommt es nur auf die letzten zwei an.

also $(9119)^n \hat{=} (\cancel{\#}19)^n \hat{=} 19^n$

somit $19^{2112} = (19^{6})^{352} = (47045881)^{352}$

auch hier nur die letzten zwei Ziffern wichtig

$81^{352} = (81^4)^{88} = (43046721)^{88}$

$21^{88} = (21^4)^{22} = (\cancel{43046721})^{22}$ 194481

$81^{22} = (21^2)^{11} = \cancel{\ldots}(6561)^{11}$

$61^{11} = 61 \cdot 61^{10} = 61 \cdot (61^5)^2 = 41 \cdot (\cancel{844596301})^2$ 844596301

$61 \cdot \cancel{01} \; 01^2 = 61$

<u>letzte 2 Ziffern</u>: $\underline{61}$

d) $\cos\beta = \frac{b^2 - a^2 - c^2}{-2ac} = \frac{1}{2} \Rightarrow \underline{\beta = 60°}$

e) $t: y = mx$; I $mx = 2^x \Rightarrow mx = \frac{m}{\ln 2}$

II $m = \ln 2 \cdot 2^x$

$2^x = \frac{m}{\ln 2}$

$\underline{x = \frac{1}{\ln 2}} \Rightarrow m = \ln 2 \cdot 2^{\frac{1}{\ln 2}} = \ln 2 \cdot e^{\ln 2 \cdot \frac{1}{\ln 2}}$

$\underline{m = e \cdot \ln 2}$