

1.

a) $f'(x) = 6x - 6k = 0$

$x = k$
 $y = -k^2$

Scheitel $(k | -k^2)$ einer nach oben offenen Parabel ist Minimum.

b) $k = -2k^2$

$2k^2 - k = 0$

$k(2k - 1) = 0$

$(k=0) \quad k = \frac{1}{2}$

c) $3(x-k)^2 - k^2 = 0$

$x - k = \pm \sqrt{\frac{k^2}{3}}$

$x_{1/2} = k \pm \frac{1}{\sqrt{3}}k$

$f'(k + \frac{1}{\sqrt{3}}k) = 6k + 2\sqrt{3}k - 6k = 2\sqrt{3}k = \tan \alpha$

$\alpha = \arctan(2\sqrt{3}k)$

2. Tangente (Symmetrie Parabel) : $\alpha_1 = \pi - \alpha$

d) $\frac{f(x_b) - f(x_a)}{x_b - x_a} = f'(x)$

$\frac{3(x-k)^2 - k^2 - (-k^2 - 3)}{x - k} = 6x - 6k$

$x = k \pm 1$
 $y = 3 - k^2$

$f'(k \pm 1) = \pm 6 = \tan \beta_{1/2} \quad \beta_{1/2} = \pm \arctan 6$

e) $\int_{x_1}^{x_2} f(x) dx = -12\sqrt{3}$

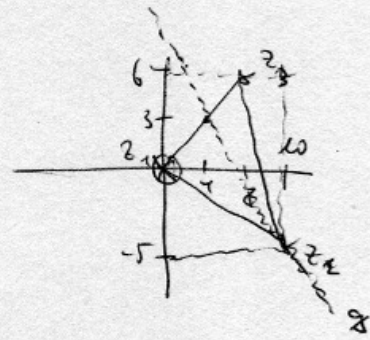
$[x^3 - 3kx^2 + 2k^2x]_{x_1}^{x_2} = 0 - 12\sqrt{3}$

$-\frac{4}{3}\sqrt{3}k^3 = -12\sqrt{3}$

$k = 3$

$$2. a) \quad \begin{array}{r} 10r + 8s = 6 \\ -50r + 6s = 2 \end{array}$$

$$r = \frac{1}{5} \quad s = \frac{1}{2}$$



EN
W12

$$b) \quad A = 14 \cdot 10 - \frac{1}{2} (6 \cdot 8 + 6 \cdot 14 + 5 \cdot 10) = 50$$

$$c) \quad z^2 = 8 + 6i \quad |z_2| = \sqrt{8^2 + 6^2} = 10$$

$$(a+bi)^2 = 8+6i$$

$$\text{I } a^2 - b^2 = 8$$

$$\text{II } 2ab = 6$$

$$a_1 = -3 \quad b_1 = -1 \quad : \quad \underline{z_1 = -3 - i}$$

$$a_2 = 3 \quad b_2 = 1 \quad : \quad \underline{z_2 = 3 + i}$$

$$d) \quad z_m = \frac{z_1 + z_2}{2} = 4 + 3i$$

$$\left. \begin{array}{l} z_1 z_2 : \begin{pmatrix} 8 \\ 6 \end{pmatrix} \\ z_m z_2 : \begin{pmatrix} 6 \\ -8 \end{pmatrix} \end{array} \right\} b$$

$$e) \quad w = f(z) = (2-i)z + 1$$

$$w_1 = f(z_1) = (2-i)(10-5i) + 1 = 16 - 20i$$

$$w_2 = f(z_2) = (2-i)(8+6i) + 1 = 25 + 4i$$

$$|2-i| = \sqrt{5}$$

$$\varphi = -26,56^\circ$$

f: Drehung um $-26,6^\circ$ (im Uhrzeigersinn) } Zentrum $z: z$
 Streckung um $\sqrt{5}$
 Verschiebung um 1 in Richtung der reellen Achse

$$z: \quad f(z) = z$$

$$(2-i)z + 1 = z$$

$$(1-i)z = -1$$

$$z = -\frac{1}{1-i} = \frac{-1(1+i)}{1+1} = -\frac{1}{2} - \frac{1}{2}i$$

$$z(-\frac{1}{2} | -\frac{1}{2}i)$$

$$3. a) \quad \alpha: 2x + y + 2z = 6$$

$$u(3|0|0) ; v(0|6|0) ; w(0|0|3)$$

$$\underline{V_{ouvw}} = \frac{1}{6} \cdot 3 \cdot 6 \cdot 3 = \underline{9}$$

$$b) \quad \text{HNF}(\alpha) \quad \frac{2x + y + 2z - 6}{3} = 0$$

$$P(1|1|3|9) \quad \left| \frac{2 \cdot 15 + 3 + 2 \cdot 9 - 6}{3} \right| = \underline{15 = d(P; \alpha)}$$

$$c) \quad g_{PQ}: \vec{x} = \begin{pmatrix} 15 \\ 3 \\ 9 \end{pmatrix} + s \begin{pmatrix} 42 \\ -15 \\ 33 \end{pmatrix} = \begin{pmatrix} 15 \\ 3 \\ 9 \end{pmatrix} + t \begin{pmatrix} 14 \\ -5 \\ 11 \end{pmatrix}$$

$$\alpha: 2 \cdot (15 + 14t) + 3 - 5t + 2(9 + 11t) = 6$$

$$t = \frac{45}{24}$$

$$\underline{D\left(\frac{795}{11} \mid -\frac{192}{11} \mid 54\right)}$$

$$d) \quad g_{PR}: \vec{x} = \begin{pmatrix} 7 \\ 7 \\ 7 \end{pmatrix} + t \begin{pmatrix} 8 \\ -4 \\ 2 \end{pmatrix} + s \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$

$$\beta \perp g: 4x - 2y + z = 0 \text{ durch } 0$$

$$\beta \cap g: 4(7 + 4s) - 2(7 - 2s) + 7 + s = 0$$

$$s = 1$$

$$\underline{S(11|15|8)}$$

$$e) \quad n: \text{ Schnitt } \alpha \text{ mit } z=0 : n = (uv) \quad \vec{x} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 3 \\ -6 \\ 0 \end{pmatrix}$$

$$\alpha: ax + by + cz = d$$

$$w(0|0|3)$$

$$\underline{3c = d}$$

$$\downarrow$$

$$\text{in var } \alpha \sim \begin{pmatrix} a \\ -2a \\ 0 \end{pmatrix}$$

$$\downarrow$$

$$ax - 2ay = 0$$

$$\underline{\alpha: 1x - 2y = 0}$$

$$4. a) 4! = 24$$

EN
H12

Bei 3 richtigen (von 4) muss die vierte auch richtig sein.

$$b) P(4) = \frac{1}{24}$$

$$P(1) = \frac{8}{24}$$

$$P(3) = 0$$

$$P(0) = \frac{9}{24}$$

$$P(2) = \frac{6}{24}$$

$$c) E(X) = 1$$

$$\sigma(X) = 1$$

$$d) P(Y=k) = \frac{1}{24} + \frac{23}{24} \cdot \frac{1}{24} + \left(\frac{23}{24}\right)^k \frac{1}{24} + \dots$$

$$= \sum_{k=0}^{\infty} \frac{1}{24} \left(\frac{23}{24}\right)^k = \frac{1}{24} \sum_{k=0}^{\infty} \left(\frac{23}{24}\right)^k$$

$$= \frac{1}{24} \frac{1 - \left(\frac{23}{24}\right)^{k+1}}{1 - \frac{23}{24}} = 1 - \left(\frac{23}{24}\right)^{k+1}$$

$$\lim_{k \rightarrow \infty} \left(1 - \left(\frac{23}{24}\right)^{k+1}\right) = 1$$

$$e) \underline{E(Y)} = 1 \cdot \frac{1}{24} + 2 \cdot \frac{23}{24} \frac{1}{24} + 3 \left(\frac{23}{24}\right)^2 \cdot \frac{1}{24} + \dots$$

$$= \frac{1}{24} \left(1 + 2 \cdot \frac{23}{24} + 3 \cdot \left(\frac{23}{24}\right)^2 + \dots\right)$$

$$= \frac{1}{24} \frac{1}{\left(1 - \frac{23}{24}\right)^2} = \underline{24}$$

$$5 a) \vec{c} \sim \begin{pmatrix} -3 \\ -2 \end{pmatrix} \text{ d.h. } c = k \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\vec{c} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 60 = 6k + 6l$$

$$k = 5$$

$$\underline{\vec{c} = \begin{pmatrix} 15 \\ 10 \end{pmatrix}}$$

(EM
W12

$$b) 3^{2x} \cdot 3 + 3^x \cdot 3^{1.5} = 4$$

$$3u^2 + 3^{1.5}u - 4 = 0$$

$$u_{1/2} = -\frac{1}{3}\sqrt{3} \quad u_2 = \frac{1}{3}\sqrt{3}$$

$$3^x = \dots < 0 \quad 3^x = 3^{-1/2}$$

↓

$$\underline{x = -1/2}$$

$$a) \binom{n+1}{n-1} = \frac{2n!}{(n+1)!} = \frac{(n+1)!}{(n-1)!(2)!} \cdot \frac{2n!(n-1)!}{(n+2)(n+1)!} = \underline{\underline{\frac{n}{n+2}}}$$

$$d) \int \underbrace{x}_{u'} \cdot \underbrace{\sin x}_{u} dx = -x \cos x + \int \cos x dx$$

$$\underline{\underline{= -x \cos x + \sin x}}$$

$$e) A \vec{x} = \vec{b}$$

$$\begin{vmatrix} 1,4-x & 0,8 \\ -0,25 & 0,5-x \end{vmatrix} = 0$$

$$\underline{\underline{x_1 = 1}}$$

$$\underline{\underline{x_2 = \frac{9}{10}}}$$

$$0,4u + 0,8v = 0$$

$$u = 2v$$

$$\underline{\underline{\vec{x}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}}}$$

$$0,5u + 0,8v = 0$$

$$u = -\frac{8}{5}v$$

$$\underline{\underline{\vec{x}_2 = \begin{pmatrix} -1,6 \\ 1 \end{pmatrix}}}$$