MATENHM

1.

a) $\vec{AB} = \begin{pmatrix} -2 \\ -4 \\ 5 \end{pmatrix}$ $\vec{AB} \times \vec{AC} = \begin{pmatrix} 5 \\ 10 \\ 10 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

$\vec{AC} = \begin{pmatrix} 0 \\ -5 \\ 5 \end{pmatrix}$ $= \vec{n}$

$E: \quad x + 2y + 2z + d = 0$

$A: \quad 4 + 4 - 6 + d = 0$

$d = -2$

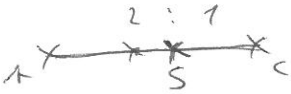
$E: x + 2y + 2z - 2 = 0$

b) $|\vec{n}| = 15$ $\underline{A = \frac{1}{2}|\vec{n}| = 7{,}5}$

$\vec{AB} \cdot \vec{AC} = 45$

$\vec{BC} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

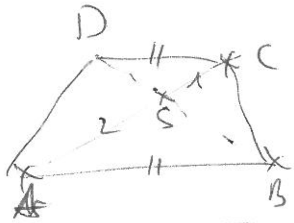
$\underline{\vec{AB} \cdot \vec{BC} = 0 \quad \boxdot}$



c) $\vec{r}_S = \frac{2\vec{r}_C + 1\vec{r}_A}{3} = \begin{pmatrix} 4 \\ -4/3 \\ 1/3 \end{pmatrix}$

$\underline{S(4 | -\frac{4}{3} | \frac{1}{3})}$

d)



Strahlensatz: $\vec{DC} = \frac{1}{2}\vec{AB} = \begin{pmatrix} -1 \\ -2 \\ 5/2 \end{pmatrix}$

$\vec{r}_D = \vec{r}_C - \vec{DC} = \begin{pmatrix} 5 \\ -1 \\ -1/2 \end{pmatrix}$

$\underline{D(5 | -1 | -\frac{1}{2})}$

d) $g: A \in g, \; g \perp E: \quad \vec{r}_g = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

$E_1: 2x + y + 2z - 38 = 0$

$HNF_1 = HNF_2$

$\pm \frac{2x + y + 2z - 38}{3} = \frac{x + 2y + 2z - 2}{3}$

(+) $x - y - 36 = 0 \quad \cap g: \quad t = -34; \quad P_1(-30 | -66 | -71)$

(−) $3x + 3y + 4z - 40 = 0 \quad \cap g: \quad t = 2: \quad P_2(6 | 6 | 1)$

MatEN H14

2. $f(x) = \frac{20x}{x^2+5}$

a) $D = \mathbb{R}$

P.S. $f(-x) = \frac{-20x}{x^2+5} = -f(x)$

NST: $x = 0$ einfach

$f'(x) = -20\,\frac{x^2-5}{(x^2+5)^2} = 0$

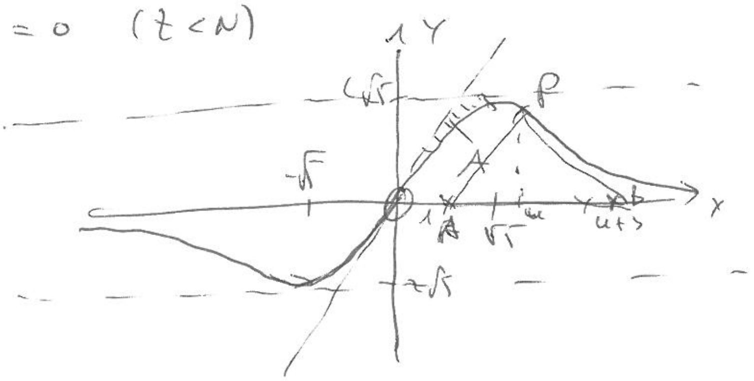
$\underline{x = \pm\sqrt{5}}$ $\quad$ $\underline{y = \pm 2\sqrt{5}}$

VZT

x	2	$\sqrt{5}$	3
$f'(x)$	$+$	0	$-$

$\nearrow \rightarrow \searrow$ Max $\underline{(\sqrt{5}\,|\,2\sqrt{5})}$ $\xrightarrow{\text{Sym}}$ Min $\underline{(-\sqrt{5}\,|\,-2\sqrt{5})}$

$\lim\limits_{|x|\to\infty} f(x) = 0$ $\quad (Z<N)$



b) $20\int \frac{x}{x^2+5}\,dx$ $\qquad x^2+5 = u$

$\frac{du}{dx} = 2x \rightarrow dx = \frac{du}{2x}$

$= 20\int \frac{x}{u}\,\frac{du}{2x} = \frac{20}{2}\int\frac{du}{u} = 10\ln u = \underline{10\ln(x^2+5)}$

c) Tangente in $(0|0)$: $y = f'(0)\cdot x = 4x$: t $\qquad t = 2\sqrt{5}$, $x = \frac{1}{2}\sqrt{5}$

$A = \int_0^{\frac{1}{2}\sqrt{5}} (t - f)\,dx + \int_{\frac{1}{2}\sqrt{5}}^{\sqrt{5}} (2\sqrt{5} - f)\,dx$

$= 10\ln\frac{4}{5} + \frac{5}{2} + 10\ln\left(\frac{4}{5}\right) + 5$

$\underline{A = 20\ln\frac{4}{5} + \frac{15}{2}}$

d) $A_{ABP_u} = \cancel{\frac{1}{2}\cdot 3\cdot f(u)}$ $\quad \frac{1}{2}(u+3-1)\cdot f(u) = \frac{1}{2}(u+2)\,\frac{20u}{u^2+5}$

$= 10\,\frac{u^2+2u}{u^2+5}$

$A'_{APB} = -20\,\frac{u^2-5u-5}{(u^2+5)^2} = 0 \Rightarrow u_{1/2} = \frac{5\pm3\sqrt{5}}{2}$

VZT $+\ 0\ -$ Max

$\underline{A(u_1) = 3\sqrt{5}+5 > 10}$

$A(0) = 0$

$A(\infty) = 10$

MatENH1 4

3. a)

(i) $p = 10 \cdot \left(\frac{1}{10}\right)^3 = 1\,\%$

(ii) $p = \frac{10 \cdot 9 \cdot 8}{1000} = 72\,\%$

b) $p = \binom{5}{2} \cdot 0{,}1^2 \cdot 0{,}9^3 = 7{,}29\,\%$

c) $P(\text{mind. eine } 7) > 99\,\%$

$1 - P(\text{nie } 7) > 0{,}99$

$0{,}9^n < 0{,}01$

$n > \log_{0{,}9} 0{,}01 = 43{,}7$

ab 44 Zahlen

d) $E = p(7\text{ teilbar}) \cdot 2 + p(3\text{ Tr.}) \cdot 1 + p(3{+}7\text{ Tr.})\, 10 - 1$

Zahlen von 00 ... 99 : 100 Stück, jede $\frac{1}{100}$

$p(7) : \frac{14}{100}$

$p(3) : \frac{33}{100}$

$p(3{+}7) : \frac{4}{100}$

$p(00) = \frac{1}{100}$

$\Big\} \frac{5}{100}$

$E = \frac{14}{100} \cdot 2 + \frac{33}{100} \cdot 1 + \frac{5}{100} \cdot 10 - 1$

$E = +0{,}11$

MatEN W14

4.1 a)

$\text{I.} \quad i z_1 + (1+i) z_2 = -3$

$\text{II.} \quad z_1 - i z_2 = 1 + 2i \quad | \cdot i$

$\text{II}'. \quad i z_1 + z_2 = -2 + i$

$\text{I} - \text{II}': \quad i z_2 = -1 - i$

$\underline{z_2 = -1 + i} \qquad \underline{z_1 = i}$

4.2 a) $z^6 = 64 \qquad z = r \cdot e^{i(\varphi + k \cdot 2\pi)}$

$\underline{r} = \sqrt[6]{64} = \underline{2}$

$\varphi_k = \frac{2\pi}{6} \cdot k \quad ; k = 0, 1, \dots 5$

$z_0 = 2 \cdot (\cos 0 + i \sin 0) = 2$

$z_1 = 2 \cdot (\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}) = 1 + i\sqrt{3}$

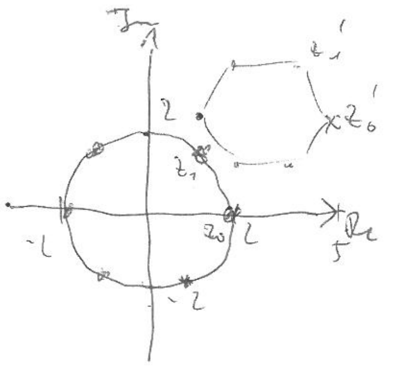
$z_2 = 2 (\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}) = -1 + i\sqrt{3}$

$z_3 = 2 (\cos \pi + i \sin \pi) = -2$

$z_4 = 2 (\cos \frac{4}{3}\pi + i \sin \frac{4}{3}\pi) = -1 - i\sqrt{3}$

$z_5 = 2 (\cos \frac{5}{3}\pi + i \sin \frac{5}{3}\pi) = +1 - i\sqrt{3}$

b)



c) reg. 6-Eck Kantenlänge $\underline{2}$

$\underline{A} = 6 \cdot \frac{2^2}{4}\sqrt{3} = \underline{6\sqrt{3}}$

d) Statt Drehung z' um M', Drehung von z um M, dann $\begin{pmatrix}3\\2\end{pmatrix}$

Drehung um 45° um (0|0): Multiplikation mit $(\frac{1}{2}\sqrt{2} + i\frac{1}{2}\sqrt{2}) = z_D$

$z_0'' = z_0 \cdot z_D + (3 + i2) = \sqrt{2} + 3 + (2 + \sqrt{2})i$

$z_1'' = z_1 \cdot z_D + \text{"} \quad = \frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2} + 3 + (2 + \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2})i$

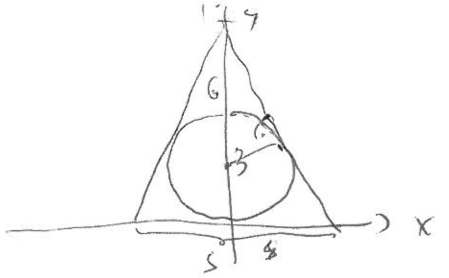
$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2} + 3 + (2 - \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2})i$

$= -\sqrt{2} + 3 + (2 - \sqrt{2})i$

$= -\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2} + 3 + (-\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2} + 2)i$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2} + 3 + (\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2} + 2)i$

5.1.



$$\frac{3}{6} = \frac{s/2}{\sqrt{6^2 - 3^2}}$$

$$\underline{\underline{s}} = \frac{1}{2}\sqrt{27} \cdot 2 = \frac{3}{2}\sqrt{3} \cdot 2 = \underline{\underline{3\sqrt{3}}}$$

5.2.

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \vec{v} = \lambda \vec{v}$$

$$(1-\lambda)(4-\lambda) - 4 = 0$$

$$\lambda_1 = 0 \qquad \vec{v}_1 \approx \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\lambda_2 = 5 \qquad \vec{v}_2 \approx \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

5.3. $2\sin^2(x) + \sin(x) - 1 = 0 \qquad [0; 2\pi]$

$$\sin x_{1/2} = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$$

$$\sin x_1 = \frac{1}{2} \qquad \sin x_2 = -1$$

$$\underline{x_1 = \frac{\pi}{3}} \qquad \underline{x_3 = \frac{3}{2}\pi}$$

$$\underline{x_2 = \frac{2}{3}\pi}$$

5.4. $f(x) = e^{ax^2+b}$

$$f'(x) = 2ax\, e^{ax^2+b}$$

I $f(1) = t(1) = 1$

II. $f'(1) = 2$

I $e^{a+b} = 1$

II. $2a\, e^{a+b} = 2$

$$2a = 2$$

$$\underline{a = 1}$$

in I: $e^{1+b} = 1$

$$\underline{b = -1}$$