ENH16

1. $f(x) = x \cdot e^{1-ax}$; $a > 0$

a) $f'(x) = e^{1-ax} + x e^{1-ax}(-a) = (1-ax)e^{1-ax} = 0$ $\quad$ VZT

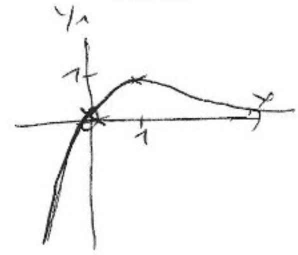
x	0	$\frac{1}{a}$	$\frac{2}{a}$
$f'(x)$	+	0	−
	↗	Max	↘

$\underline{x = \frac{1}{a}}$ $\underline{y = \frac{1}{a}}$

$\lim_{x \to +\infty} \underbrace{x}_{+\infty} \underbrace{e^{1-ax}}_{0} = 0$ $\quad$ pos. x-Achse ist waag. A.

$\lim_{x \to -\infty} \underbrace{x}_{-\infty} \underbrace{e^{1-ax}}_{\infty} = -\infty$

$a = 1$



b) $f''(x) = a\underbrace{(ax-2)}\, e^{1-ax} = 0$

$\underline{x = \frac{2}{a}}$ $\underline{y = \frac{2}{ea}}$ $\quad f'(\frac{2}{a}) = -\frac{1}{e}$

$t: \underline{y = -\frac{1}{e}(x - \frac{2}{a}) + \frac{2}{ae} = -\frac{1}{e}x + \frac{4}{ae}}$

c) $H(\frac{1}{a}|\frac{1}{a})$ $W(\frac{2}{a}|\frac{2}{ea})$ $\quad \underline{m} = \frac{\frac{2}{ea} - \frac{1}{a}}{\frac{2}{a} - \frac{1}{a}} = \frac{\frac{2-e}{ea}}{\frac{1}{a}} = \underline{\frac{2-e}{e}}$ unabhängig von a, also parallel

d) $A = 1 = \int_0^\infty f(x)dx = \left[-\frac{1}{a^2}(ax+1)e^{1-ax}\right]_0^\infty = +\frac{e}{a^2} \stackrel{!}{=} 1$

$\underline{a = \sqrt{e}}$

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2. 3 x '1' ; 2 x '2' ; 1 x '3' $\quad$ 10 Würfe

a) $P(10 \times '1') = (\frac{1}{2})^{10} = \frac{1}{1024} = \underline{0{,}0977\%}$

b) $P(\text{kein } '3') = (\frac{5}{6})^{10} = \underline{16{,}15\%}$

c) $P(\text{mind. ein } '3') = 1 - P(\text{kein } '3') = \underline{83{,}8\%}$

d) $P(3 \times '1') = \binom{10}{3}(\frac{1}{2})^3(\frac{1}{2})^7 = \frac{15}{128} = \underline{11{,}7\%}$

e) $P(\text{mind. zweimal } '2')$

$= 1 - P(\text{keinmal } '2') - P(\text{einmal } '2')$

$= 1 - (\frac{4}{6})^{10} - 10 \cdot (\frac{2}{6})^1(\frac{4}{6})^9 = \underline{89{,}6\%}$

f) $P(\text{mind. einmal } '3') > 0{,}8$

$1 - P(\text{kein } '3') > 0{,}8$

$1 - (\frac{5}{6})^n > 0{,}8$

$n > \log_{\frac{5}{6}} 0{,}2 = 8{,}827$ $\quad$ \underline{ab 9 Würfen}

~~g)~~ 3 Würfel, Augensumme

3	6	9	4	5	5	7	7	8	6
111	222	333	112	113	221	223	331	332	123
0,125	0,037	0,00463	0,25	0,125	0,166	0,055	0,00694	0,00463	0,166

g) Erwartungswert

$E(AS) = 3 \cdot 0{,}125 + 4 \cdot 0{,}25 + 5 \cdot 0{,}2916 + 6 \cdot 0{,}20366 + 7 \cdot 0{,}0625 + 8 \cdot 0{,}00463 + 9 \cdot 0{,}00463 = 4{,}57$

h) $P(AS > 7) = 0{,}07175$ $\quad E = 6 \cdot 0{,}07175 - 1 \cdot 0{,}92825 = -0{,}5$ $\quad$ nicht fair

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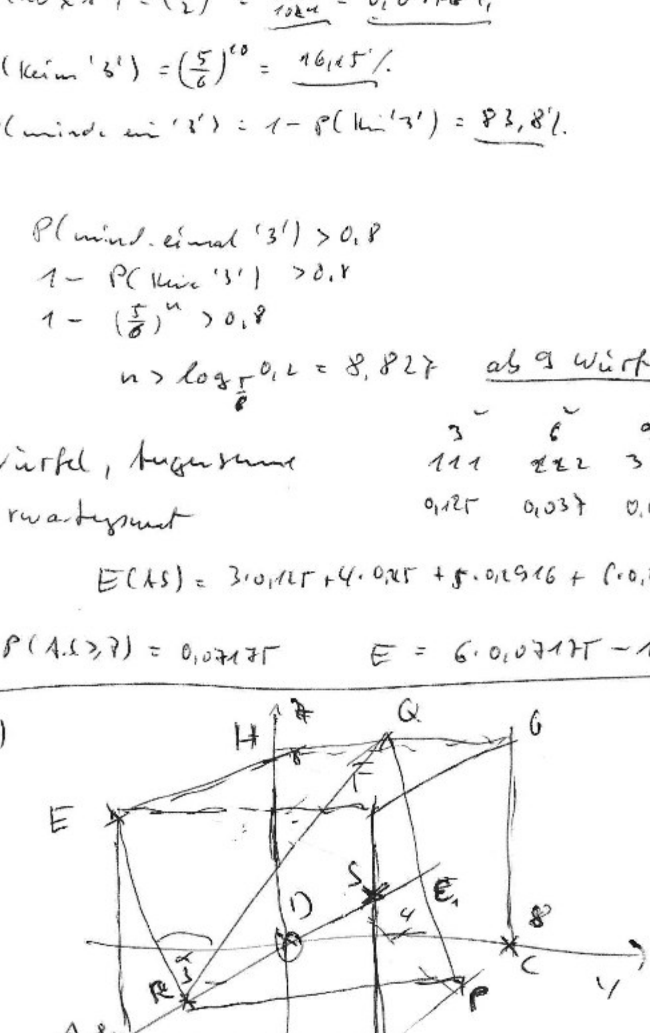
3. a)



$E_1: \vec{x} = \begin{pmatrix} 8 \\ 0 \\ 6 \end{pmatrix} + s\begin{pmatrix} -3 \\ 8 \\ 0 \end{pmatrix} + t\begin{pmatrix} -5 \\ 4 \\ 8 \end{pmatrix}$

$\quad\quad\quad\quad\quad\quad\quad RP \quad\quad\quad RQ$

$\vec{n}_1 = \overline{RP} \times \overline{RQ} = \begin{pmatrix} 64 \\ 16 \\ 32 \end{pmatrix} \sim \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$

$E_1: 4x + y + 2z + d = 0$

$B(0|0|5)$: $\quad d = -10$

$\underline{E_1: 4x + y + 2z - 20 = 0}$

b) $\vec{n}_2 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ $\quad \vec{n}_2 \cdot \vec{n}_1 = 0$ $\quad \perp$

I. $4a + b + 2c = 0$

$Q(0|4|8)$ $\quad$ II $\quad 4b + 8c + d = 0$

$S(8|8|4)$ $\quad$ III $\quad 8a + 8b + 4c + d = 0$

$a = \frac{1}{16}d \quad b = -\frac{1}{6}d \quad c = -\frac{1}{24}d \quad | \cdot 48 = d$

$\underline{E_2: 3x - 8y - 2z + 48 = 0}$

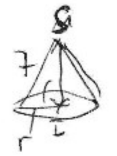
c) $\cos\alpha = \frac{\vec{RQ}\cdot\vec{RE}}{RQ\ RE} = \frac{\begin{pmatrix}-5\\4\\8\end{pmatrix}\cdot\begin{pmatrix}3\\0\\8\end{pmatrix}}{\sqrt{105}\ \sqrt{73}} = \frac{49}{\sqrt{5}} \approx 0{,}5591 \Rightarrow \underline{\alpha \approx 55{,}7^\circ}$

d) $A = \frac{1}{2}|\underbrace{\vec{RP}\times\vec{RQ}}_{\vec{n}_1}| = \frac{1}{2}\left|\begin{pmatrix}-8\\8\\0\end{pmatrix}\times\begin{pmatrix}3\\0\\8\end{pmatrix}\right| = \frac{1}{2}\left|\begin{pmatrix}64\\-16\\32\end{pmatrix}\right| = \underline{8\sqrt{21}}$

f) $g:\ \vec{x} = \begin{pmatrix}8\\8\\4\end{pmatrix} + s\begin{pmatrix}4\\1\\2\end{pmatrix} \cap E_1 \qquad 4(8+4s) + 8+s + 2(4+2s) - 20 = 0$

$\underline{s = -\frac{4}{3}} \qquad \underline{I\left(\frac{8}{3}\middle|\frac{20}{3}\middle|\frac{4}{3}\right)}$

g) $K(S;7) \cap E_1 \to k:\ r?$



$\vec{SI} = -\frac{4}{3}\begin{pmatrix}4\\1\\2\end{pmatrix} \qquad SI = \frac{4}{3}\sqrt{21}$

$r = \sqrt{7^2 - SI^2} = \sqrt{7^2 - \left(\frac{4}{3}\sqrt{21}\right)^2} = \underline{\frac{1}{3}\sqrt{105}}$

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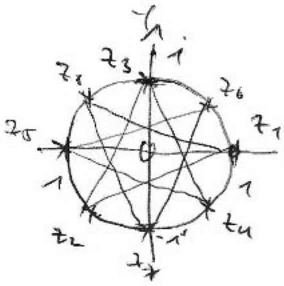
4. a) $z_{n+1} = z_n\cdot\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) \quad z_1 = 1$

$z_2 = -\frac{1}{2}\sqrt{2}(1+i)$

$z_3 = i$

$z_4 = \frac{1}{2}\sqrt{2}(1-i)$



$z_{n+1} = z_n\cdot \mathrm{cis}\left(\frac{5}{4}\pi\right)$

$z_2 = \mathrm{cis}\left(\frac{5}{4}\pi\right)$

$z_3 = \left[\mathrm{cis}\left(\frac{5}{4}\pi\right)\right]^2 = \mathrm{cis}\left(\frac{5}{2}\pi\right) = \mathrm{cis}\left(\frac{\pi}{2}\right)$

$z_4 = [\ \ "\ \ ]^3 = \mathrm{cis}\left(\frac{15}{4}\pi\right) = \mathrm{cis}\left(-\frac{\pi}{4}\right) = \mathrm{cis}\left(\frac{7}{4}\pi\right)$

$z_n = \left[\mathrm{cis}\left(\frac{5}{4}\pi\right)\right]^{n-1} = 1$

$\left(\frac{5}{4}\pi\right)\cdot(n-1) = 2\pi\cdot k$

$n = \frac{8k}{5} + 1 \qquad k = 5$

$\underline{n = 9}$

$|z_1 z_2| = \sqrt{\left(\frac{1}{2}\sqrt{2}\right)^2 + \left(1 + \frac{1}{2}\sqrt{2}\right)^2} = \sqrt{2+\sqrt{2}}$

$\underline{s} = 8\cdot|z_1z_2| = \underline{8\cdot\sqrt{2+\sqrt{2}}}$

b) $M = \frac{1}{2}\begin{pmatrix}1 & 3\\ 3 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} \qquad Mx = \lambda x$

$(M - \lambda 1)x = 0$

$\frac{1}{2}\begin{vmatrix}1-2\lambda & 3\\ 3 & 1-2\lambda\end{vmatrix} = 0 \qquad \begin{pmatrix}3 & 3\\ 3 & 3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} =$

$(1-2\lambda)^2 - 9 = 0$

$\underline{\lambda_1 = -1} \qquad \underline{\vec{v}_1 = \begin{pmatrix}-1\\1\end{pmatrix}}$

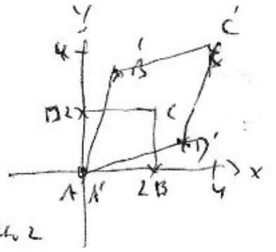
$\underline{\lambda_2 = 2} \qquad \underline{\vec{v}_2 = \begin{pmatrix}1\\1\end{pmatrix}}$

$A(0|0) \to A'(0|0)$

$B(2|0) \to B'(1|3)$

$C(2|2) \to C'(4|4)$

$D(0|2) \to D'(3|1)$



$\vec{v}_2 \mathrel{\hat{=}} C$: also nur Streckung um Faktor 2

z.B. $E(-4|4) \to E'(4|-4)$ Spiegelung $\lambda(-1)$

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5. a) $6u^2 + u - 2 = 0 \qquad u_{1/2} = -\frac{2}{3} \,\middle|\, \frac{1}{2}$

$\frac{1}{2}:\ x = \frac{\pi}{6} \,\middle|\, \frac{5}{6}\pi + z\cdot 2\pi$

$-\frac{2}{3}:\ x = -0{,}73 \,|\, 3{,}871 + z\cdot 2\pi$

in $[-\pi; 3\pi]$

b) $(3x-6)^2 + (2y+6)^2 = 36$

$\frac{(x-2)^2}{3^2} + \frac{(y+3)^2}{2^2} = 1$

$a = 3$

$b = 2$

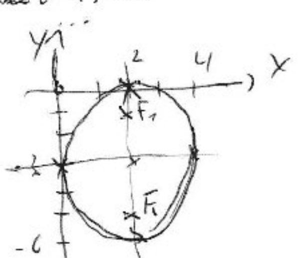
$c = \sqrt{a^2 - b^2} = \sqrt{5}$

$x-2$, $y+3$: 2 nach rechts, 3 nach unten

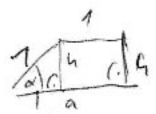
$F_1(2|+\sqrt{5}-3)$

$F_2(2|-\sqrt{5}-3)$

F



c)



$\left(\tan\alpha = \frac{h}{a-1}\right)$

$h = 1\cdot\cos\alpha$

$a - 1 = \cos\alpha$

$A = \frac{a+1}{2}\cdot h \to \max$

$= \frac{\cos\alpha + 2}{2}\cdot\sin\alpha$

$A = \frac{1}{2}(\sin\alpha\cos\alpha + 2\sin\alpha) \qquad \alpha\in[0;90°]$

$A' = \frac{1}{2}(\cos^2\alpha - \sin^2\alpha + 2\cos\alpha)$

$= \frac{1}{2}(2\cos^2\alpha - 1 + 2\cos\alpha) = 0$

$\cos\alpha = \frac{-1\pm\sqrt{3}}{2}$

$\cos\alpha \approx 0{,}366$

$\underline{\alpha \approx 68{,}5^\circ}$

α	60°	68,5°	70°
A'	+	0	−

↗ Max ↘

$A(0°) = 0$

$A(90°) = 1$

$\underline{A(68{,}5°) = 1{,}1}$