ENV H17

1. $f_a(x) = a \sin(ax)$

a) $ax = \pi + z \cdot 2\pi$

NST $\underline{x = \frac{\pi}{a} + z \cdot \frac{2\pi}{a}}$

HP: $ax = \frac{\pi}{2}$

$\underline{x = \frac{\pi}{2a}}$ $\quad y = a \quad HP(\frac{\pi}{2a} | a)$

b) $x = \frac{\pi}{2a} \Rightarrow a = \frac{\pi}{2x}$

$y = a \rightarrow \underline{y} = \frac{\pi}{2x} = \underline{\frac{\pi}{2} \cdot \frac{1}{x}}$ $\quad x > 0$, da $a > 0$

c) $\int_0^{\frac{\pi}{a}} f_a(x) = 2$

$[-\cos(ax)]_0^{\frac{\pi}{a}} = 2$

$1 + 1 = 2$ ✓ immer erfüllt, a spielt keine Rolle, da sowohl Streckung in y-Richtung mit a, als auch Stauchung in x-Richtung mit a.

d) $\pi \int_0^{\frac{\pi}{a}} (a \sin(ax))^2 dx = \pi \left[a^2 \left(\frac{x}{2} - \frac{\sin(2ax)}{4a} \right) \right]_0^{\frac{\pi}{a}} = \frac{\pi^2}{2} a$

e) $f_2(x) = 2\sin(2x)$

$y = ax^2 + bx + c = a \cdot (x - \frac{\pi}{2}) \cdot x$ $\quad y(\frac{\pi}{4}) = 2$

$a(\frac{\pi}{4} - \frac{\pi}{2}) \cdot \frac{\pi}{4} = 2$

$\underline{a = -\frac{32}{\pi^2}}$

$y = -\frac{32}{\pi^2}(x - \frac{\pi}{2})x$

2. $z_n = (\frac{3}{4} i)^n \; ; \; n \in \mathbb{N}$

a) $z_1 = \frac{3}{4} i \; ; \; z_2 = -\frac{9}{16} \; ; \; z_3 = -\frac{27}{64} i \; ; \; z_4 = \frac{81}{256}$

b) $z_1 = a + bi$ $\quad z_1 \cdot z_2 = a \cdot c - bd + (ad + bc)i$

$z_2 = c + di$

$\left.\begin{array}{l} |z_1 z_2|^2 = (ac - bd)^2 + (ad + bc)^2 = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2 \\ |z_1|^2 |z_2|^2 = (a^2 + b^2)(c^2 + d^2) = \quad '' \end{array}\right\} = ✓$

c) $d_n = z_{n+1} - z_n$

$d_0 = z_1 - z_0 = \frac{3}{4} i + \frac{9}{16}$ $\quad \underline{|d_0| = \sqrt{(\frac{3}{4})^2 + (\frac{9}{16})^2}} = \underline{\frac{15}{16}}$

$d_1 = z_2 - z_1 =$ ~~...~~

$= -\frac{27}{64} i + \frac{9}{16}$ $\quad \underline{|d_1| = \sqrt{(\frac{27}{64})^2 + (\frac{9}{16})^2}} = \underline{\frac{45}{64}}$

$\Big\} \cdot \frac{3}{4} = q$

$\underline{\sum_{n=0}^{\infty} |d_n|^2} = \frac{15}{16} \sum_{n=0}^{\infty} (\frac{3}{4})^n = \frac{15}{16} \cdot \frac{1}{1 - \frac{3}{4}} = \underline{\underline{\frac{15}{4}}}$

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2, d) $r(\varphi) = \left(\frac{3}{4}\right)^n \quad n = \frac{\varphi}{\frac{\pi}{2}} = \frac{2\varphi}{\pi}$

$$r(\varphi) = \left(\frac{3}{4}\right)^{\frac{2\varphi}{\pi}}$$

e) $A = \frac{1}{2}\int_0^{2\pi} \left(\frac{3}{4}\right)^{\frac{2\varphi}{\pi}} d\varphi = \frac{1}{2}\left[\frac{\pi}{2\ln\left(\frac{3}{4}\right)} \cdot \left(\frac{3}{4}\right)^{\frac{2\varphi}{\pi}}\right]_0^{2\pi}$

$$= -\frac{175\pi}{1024\ln(3/4)} = \frac{175\pi}{1024\ln(4/3)} \approx 1{,}866$$

3, a) $\vec{AP} \cdot \vec{AB} = 0$

$$\begin{pmatrix} 16 \\ -8 \\ p_3 - 3 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \\ 8 \end{pmatrix} = 0$$

$8p_3 - 40 = 0$

$\underline{p_3 = 5}$ $\quad \underline{P(17|-4|5)}$

$\cancel{AB^2 = AB^2}$

$\vec{AP} = \begin{pmatrix} 16 \\ -8 \\ 2 \end{pmatrix} \quad AP^2 = 324 \quad AB = \frac{1}{2}AP$

$AB^2 = 81 \quad \vec{AD} = \frac{1}{2}\vec{AP}$

$\vec{D} = \frac{1}{2}\vec{AP} + \vec{A} = \begin{pmatrix} 9 \\ 0 \\ 4 \end{pmatrix} \quad \underline{D(9|0|4)}$

b) $\vec{n} \cancel{\vec{AE}} = \vec{AB} \times \vec{AD} = \begin{pmatrix} 36 \\ 63 \\ -36 \end{pmatrix} \quad \vec{E} = \vec{AE} + A = \begin{pmatrix} 5 \\ 11 \\ -1 \end{pmatrix} \quad E\left(\cancel{37}\middle|\cancel{64}\middle|\cancel{-33}\right) \; E(5|11|-1)$

~~ist~~ $|\vec{n}| = 81 = AB$

also $\vec{n} = \vec{AE} \cdot 9$

$\vec{AE} = \frac{1}{9}\vec{n} = \begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix}$

c) $\vec{M} = \vec{A} + \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AE} = \begin{pmatrix} 7/2 \\ 15/2 \\ 5 \end{pmatrix} \quad \underline{K: (x-\frac{7}{2})^2 + (y-\frac{15}{2})^2 + (z-5)^2 = \frac{18683}{4}}$

$R = \frac{1}{2}AB\cdot\sqrt{3} = \frac{81}{2}\sqrt{3}$

d) $\vec{n} = \vec{MA} = \begin{pmatrix} -5/2 \\ -11/2 \\ -2 \end{pmatrix} \sim \begin{pmatrix} 5 \\ 11 \\ 4 \end{pmatrix} \quad E: 5x + 11y + 4z + d = 0$

$A(1|4|3): \; 5 + 44 + 12 + d = 0$

$\underline{E: 5x + 11y + 4z - 61 = 0}$

e) $\vec{H} = \vec{E} + \vec{AD} = \begin{pmatrix} 13 \\ 7 \\ 0 \end{pmatrix}$

$\cos\varphi = \frac{\vec{PH}\cdot\vec{PF}}{PH \; PF} = \frac{\begin{pmatrix} -4 \\ 11 \\ -5 \end{pmatrix}\begin{pmatrix} -11 \\ 15 \\ 2 \end{pmatrix}}{\sqrt{162}\sqrt{486}} = \frac{243}{162\sqrt{3}} = \frac{1}{2}\sqrt{3}$

$F = \vec{B} + \vec{AE} = \begin{pmatrix} 6 \\ 15 \\ 7 \end{pmatrix}$

$\underline{\varphi = 30°}$

4. a) P(mind. eins von zwei nicht bin h) = 0,9775

$1 - P(\text{beide bin h}) = 0{,}9775$

$1 - p^2 = 0{,}9775$

$\underline{p = 15\%}$

b) $P(3 \text{ von } 12 \to 7) = \binom{12}{3} p^3 (1-p)^9$

$= \frac{12!}{3!\,9!} 0{,}15^3\, 0{,}85^9 = \underline{17{,}2\%}$

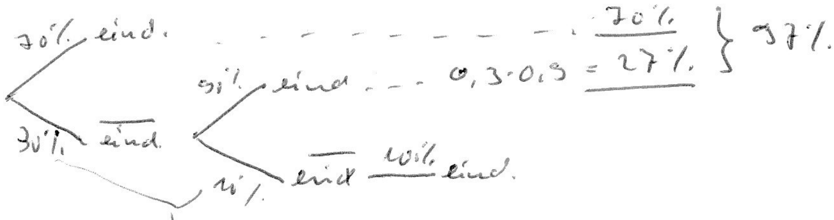
c) $\underline{(1-p)^8}$: WS, alle nicht nach 7.

$1-p^8$: WS mind. eines nicht nach 7.

d) SS: 70%

3S: 90%

D: 100%



e) $\varnothing = 0{,}7 \cdot 20s + 0{,}3 \cdot 70s + 0{,}03 \cdot 370s = \underline{46{,}1s}$

5 a) $\log n = 2017 \cdot \log 2017 = 2017 \cdot 3{,}34.. = 6665{,}591957$

$n = 10^{6665{,}59..} = 10^{6665} \cdot 10^{0{,}59} = 3{,}9065.. \cdot 10^{6665}$

6666 Stellen; erste Ziffer 3906..

letzte Ziffer ~ 7^{2017}

n	1	2	3	4	5	6	…	2015	2016	2017
7^n	7	9	3	1	7	9	…	7	9	$\underline{\underline{3}}$

b) $g(x) = 3 \sin\left[\frac{1}{3}(x-5)\right] + 7$

c) $5u + 3 = \lambda u$ $\quad 5u+3 = (6u+2)u \Rightarrow \quad u_1 = 1 \quad \lambda_1 = 8$

$6u + 2 = \lambda$ $\quad\quad u_2 = -\frac{1}{2} \quad \lambda_2 = -1$

d) $f' = 4ax^3 + 3bx^2 + 2cx + d$

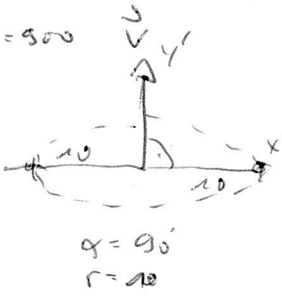
$f'' = 12ax^2 + 6bx + 2c$ $\quad D = (6b)^2 - 4 \cdot 12a \cdot 2c < 0$

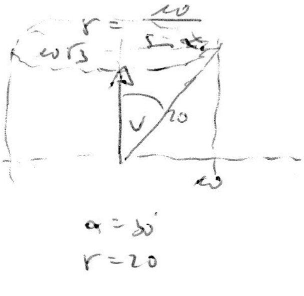
$\underline{3b^2 - 8ac < 0}$

e) $\left|\vec{r} \times \underbrace{\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}}_{\vec{v}}\right|^2 = 900$ $\quad |\vec{r} \times \vec{v}| = r \cdot v \cdot \sin\alpha = 30$

$v = 3$ $\quad r \sin\alpha = 10$ $\quad x' = r \cdot \sin\alpha = 10$

$5x^2 + 8y^2 + 5z^2 - 4xy - 8xz - 4yz = 900$ $\quad r = \frac{10}{\sin\alpha}$ $\quad y' = r \cdot \cos\alpha = 10 \cot\alpha$





Zylinder

Achse $\vec{v}$

Radius 10