EN lt19

1 a) $\begin{pmatrix}2\\1\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix} = 2+2-4 = 0 \quad \perp$

b) $A \in h$? $\begin{pmatrix}5\\-7\\7\end{pmatrix} = \begin{pmatrix}7\\-3\\11\end{pmatrix} + s\begin{pmatrix}1\\2\\2\end{pmatrix}$ $\quad \underline{s=-2}$

$k: \begin{pmatrix}2\\1\\-2\end{pmatrix} \times \begin{pmatrix}1\\2\\2\end{pmatrix} = \begin{pmatrix}6\\-6\\3\end{pmatrix} \sim \begin{pmatrix}2\\-2\\1\end{pmatrix}$

$k: \vec{X} = \begin{pmatrix}5\\-7\\7\end{pmatrix} + t\begin{pmatrix}2\\-2\\1\end{pmatrix}$

$\vec{u} = \begin{pmatrix}-1\\-1\\4\end{pmatrix} \quad \vec{v} = \begin{pmatrix}1\\2\\2\end{pmatrix}$

$\overline{AB} = \begin{pmatrix}-1\\-7\\-4\end{pmatrix} - \begin{pmatrix}7\\-3\\11\end{pmatrix} = \begin{pmatrix}-8\\-4\\7\end{pmatrix}$

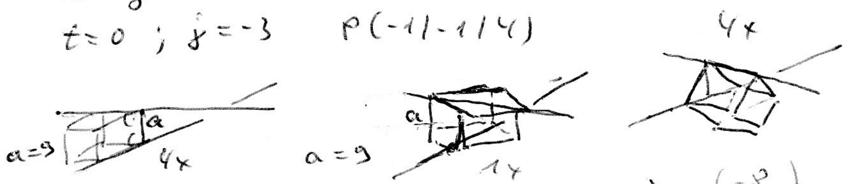
Abstand (g,h) $d = \dfrac{|\overline{AB}\cdot(\vec{u}\times\vec{v})|}{|\vec{u}\times\vec{v}|}$

$d = 9$

c) $k \cap g$

$t=0$; $s=-3$ $\quad P(-1|-1|4)$

d)



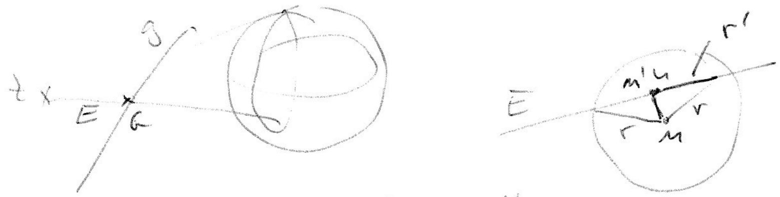
$a\sqrt{2} = d$

$a = \frac{9}{2}\sqrt{2}$

e) $r = \sqrt{117} \quad T(-1|-1|4) \quad \overrightarrow{MT} = \begin{pmatrix}-8\\2\\-7\end{pmatrix} \quad \overline{MT} = \sqrt{117}$ } Berührung

$\overrightarrow{MT}\cdot\begin{pmatrix}2\\1\\-2\end{pmatrix} = -16+2+14 = 0$

f)



$E: \vec{X} = \begin{pmatrix}-5\\-11\\-10\end{pmatrix} + t\begin{pmatrix}2\\1\\-2\end{pmatrix} + s\begin{pmatrix}4\\11\\14\end{pmatrix} \qquad \begin{pmatrix}2\\1\\-2\end{pmatrix} \times \begin{pmatrix}4\\11\\14\end{pmatrix} = \begin{pmatrix}36\\-36\\18\end{pmatrix} \sim \begin{pmatrix}2\\-2\\1\end{pmatrix}$

$2x - 2y + z + d = 0$

$d = -4$

$E: 2x - 2y + z - 4 = 0$

$M(7|-3|11) \quad h = \left|\dfrac{2\cdot 7 - 2\cdot(-3) + 11 - 4}{3}\right| = \dfrac{27}{3} = 9$

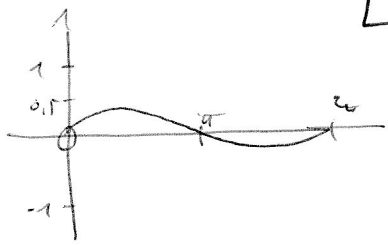
$\underline{r' = \sqrt{r^2 - h^2}} = \underline{6}$

$\overline{MM'} = -\begin{pmatrix}2\\-2\\1\end{pmatrix}\cdot \underset{\frac{9}{3}}{3} = \begin{pmatrix}-6\\6\\3\end{pmatrix} \qquad \vec{r}_{M'} = \vec{r}_M + \overline{MM'} = \begin{pmatrix}7\\-3\\11\end{pmatrix} + \begin{pmatrix}-6\\6\\3\end{pmatrix} = \begin{pmatrix}1\\3\\14\end{pmatrix}$

$\underline{M'(1|3|14)}$

LENH19

2. $f(x) = e^{-\frac{1}{\sqrt{3}}x} \sin x$



a) $f'(x) = \left(\cos x - \frac{1}{3}\sqrt{3}\sin x\right) e^{-\frac{1}{\sqrt{3}}x}$

$f''(x) = -\frac{2}{3}\left(2\cos x + \sin x\right) e^{-\frac{1}{\sqrt{3}}x}$

NST: $\sin x = 0$ $\quad \underline{x = 0 + z\cdot\pi}$ $\quad z \in \mathbb{N}_0$

Ext: $\tan x = \sqrt{3}$ $\quad \underline{x = \frac{\pi}{3} + z\cdot\pi}$ $\quad$ ~~a~~ $y = (-1)^z \cdot e^{-\frac{1}{\sqrt{3}}\left(\frac{\pi}{3} + z\pi\right)}$ $\quad y_1 = 0{,}47$, $y_2 = -0{,}077$

WP: $\tan x = -2$ $\quad x = 2{,}03 + z\cdot\pi$

b) $f'(\pi) = -e^{-\frac{\pi}{\sqrt{3}}}$ $\quad t:\ y = -e^{-\frac{\pi}{\sqrt{3}}}(x-\pi) + 0$

$= \tan\alpha \Rightarrow (\alpha = -9{,}3°) \rightarrow$ y-Achse: $\underline{\underline{80{,}7°}}$

c) $\int e^{-\frac{x}{\sqrt{3}}} \sin x \, dx = -\sqrt{3} e^{-\frac{x}{\sqrt{3}}} \sin x - \int -\sqrt{3} e^{-\frac{x}{\sqrt{3}}} \cos x \, dx$

$= -\sqrt{3} e^{-\frac{x}{\sqrt{3}}} \sin x + \sqrt{3}\left(-\sqrt{3} e^{-\frac{x}{\sqrt{3}}} \cos x - \int -\sqrt{3} e^{-\frac{x}{\sqrt{3}}} (-\sin x)\, dx\right)$

$= -\sqrt{3} e^{-\frac{x}{\sqrt{3}}} \sin x - 3 e^{-\frac{x}{\sqrt{3}}} \cos x - 3\int e^{-\frac{x}{\sqrt{3}}} \sin x \, dx$

$4 \cdot \int e^{-\frac{x}{\sqrt{3}}} \sin x \, dx = \quad " \quad "$

$\int e^{-\frac{x}{\sqrt{3}}} \sin x \, dx = -\frac{1}{4}\left(3\cos x + \sqrt{3}\sin x\right) e^{-\frac{x}{\sqrt{3}}}$

$= -\frac{\sqrt{3}}{4}\left(\sqrt{3}\cos x + \sin x\right) e^{-\frac{x}{\sqrt{3}}}$

d) $x = n\cdot\pi$: $\sqrt{3}\cos x + \sin x = \pm\sqrt{3}$ d.h. $\left|-\frac{\sqrt{3}}{4}\left(\sqrt{3}\cos x + \sin x\right)\right| = \frac{3}{4}$

$A_n = F((n+1)\pi) - F(n\cdot\pi) = \frac{3}{4} e^{-\frac{n+1}{\sqrt{3}}\pi} + \frac{3}{4} e^{-\frac{n\pi}{\sqrt{3}}}$

$= \frac{3}{2} e^{-\frac{\pi}{\sqrt{3}}} \cdot e^{-\frac{n}{\sqrt{3}}\pi} = \frac{3}{2} e^{-\frac{\pi}{\sqrt{3}}} \cdot q^n$ $\quad q = e^{-\frac{\pi}{\sqrt{3}}}$

$\sum_{n=0}^{\infty} A_n = \frac{3}{2} e^{-\frac{\pi}{\sqrt{3}}} \cdot \frac{1}{1-q} = \frac{3}{2} e^{-\frac{\pi}{\sqrt{3}}} \cdot \frac{1}{1 - e^{-\pi/\sqrt{3}}} = \frac{3}{2} \frac{1}{e^{\pi/\sqrt{3}} - 1}$

EN 11/9

3. 4x 20 : 6 r ; 4 b ; 10 g

a) 3 ein Griff

a_1) WS(3 versch. F.) = 3! $\frac{6 \cdot 4 \cdot 10}{20 \cdot 19 \cdot 18}$ = 21,05 %

a_2) WS(3 rote) = $\frac{6 \cdot 5 \cdot 4}{20 \cdot 19 \cdot 18}$ = 1,75 %

a_3) WS(mind. ein blau) = 1 − (kein blau) = $1 - \frac{\overset{15 \cdot 14}{16 \cdot 9 \cdot 8}}{20 \cdot 19 \cdot 18}$ = 50,9 %

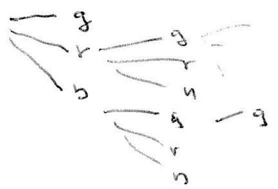
b) WS(min. ein grün) > 0,95

1 − WS(kein grün) > 0,95

ab 4

c) $\frac{20!}{6!\,4!\,10!} = 38798760$

d)



d_1) r r : $\frac{6 \cdot 5}{20 \cdot 19}$ = 7,89 %

d_2) $E = 0 \cdot \frac{10}{20} + 1\left(\frac{1}{2} + \frac{1}{4}\right) + 2\left(\frac{1}{4} + \frac{1}{8}\right) + 3\,\frac{1}{16} =$ 2,7

d_3) r r g
r b g
b r g
b b g

$2 \cdot \frac{6 \cdot 4 \cdot 10}{20 \cdot 19 \cdot 18}$ 7,01 %

$\frac{6 \cdot 5 \cdot 10}{20 \cdot 19 \cdot 18}$

$\frac{4 \cdot 3 \cdot 10}{20 \cdot 19}$

13,16 %

$\frac{7,01}{13,16}$ = 53 %

ENUA

4. I a)

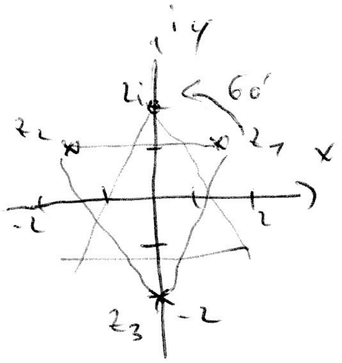
$z^3 = 8i \qquad i : \frac{\pi}{2} \qquad + k \cdot \frac{2\pi}{3}$

$\sqrt[3]{8} = 2 \qquad \frac{\pi}{2} : 3 = \frac{\pi}{6}$

$z_1 = 2 \cdot (\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}) = 2 \left(\frac{1}{2}\sqrt{3} + i \frac{1}{2}\right) = \underline{\sqrt{3} + i}$

$z_2 = 2 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right) = 2\left(-\frac{1}{2}\sqrt{3} + i\frac{1}{2}\right) = \underline{-\sqrt{3} + i}$

$z_3 = 2 \left(\cos \frac{9\pi}{6} + i \sin \frac{9\pi}{6}\right) = 2(0 - i) = \underline{-2i}$



b) $z_1' = 2i$

$z_2' = -\sqrt{3} - 1i \qquad z^3 = -8i$

$z_3' = \sqrt{3} - i$

II. $\vec{e}_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad \vec{e}_y = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

a) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -2 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -3 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad A = 1$

c) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & -3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 2a - 2b & -3a + b \\ 2c - 2d & -3c + d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$2a - 2b = 1$
$-3a + b = 0$
$2c - 2d = 0$
$-3c + d = 1$

$a = -\frac{1}{4}$
$b = -3/4$
$c = -1/2$
$d = -1/2$

$A^{-1} = -\frac{1}{4}\begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$

d) $\begin{pmatrix} 2 & -3 \\ -2 & 1 \end{pmatrix} \vec{v} = \lambda v \to \begin{pmatrix} 2-\lambda & -3 \\ -2 & 1-\lambda \end{pmatrix} \vec{v} = 0$

$(2-\lambda)(1-\lambda) - 6 = 0$

$\lambda_1 = -1 \qquad \vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = 4 \qquad \vec{v}_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

e) $g_1 : \vec{x} = \vec{r}_p + s\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$g_2 : \vec{x} = \vec{r}_a + t\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$\vec{r}_{p,a}$ beliebig

CEN1/19

5.

I. $A = \frac{1+1+2\cos a}{2} \cdot \sin a = (1+\cos a)\sin a$

$A' = 2\cos^2 x + \cos x - 1 = 0$

$\cos x = -1 \rightarrow 180°$

$\cos x = \frac{1}{2} \rightarrow 60°$

$A'' = -4\sin x \cos x - \sin x \,|_{60°} < 0 \rightarrow \text{Max}$

II. $a_1 = 3 \quad a_{n+1} = 2a_n - 1$

$a_2 = 5$ $+2$

$a_3 = 9$ $+4$

$a_4 = 17$ $+8$

$a_n = 1 + 2^n$

$$\left.\begin{aligned} a_{n+1} &= 1 + 2^{n+1} \\ 2a_n - 1 &= 2\cdot(1+2^n) - 1 \\ &= 2 + 2^{n+1} - 1 \\ &= 2^{n+1} + 1 \end{aligned}\right] = ✓$$

III. $25x^2 + 9y^2 + 100x - 54y - 44 = 0$

$25(x^2 + 4x + 4 - 4) + 9(y^2 - 6y + 36 - 36) = 44$

$25(x+2)^2 - 100 + 9(y-3)^2 - 324 = 44$

$25(x+2)^2 + 9(y-3)^2 = 468$

Ellipse: $M(-2|3)$ Endpunkte: $(-2|-2)$; $(-2|8)$; $(-5|3)$; $(1|3)$

$a = 10/2 = 5$
$b = 6/2 = 3$

$e = \sqrt{a^2 - b^2} = 4$ Brennpunkt: $(-2|-1)$ □ $(-2|7)$ □