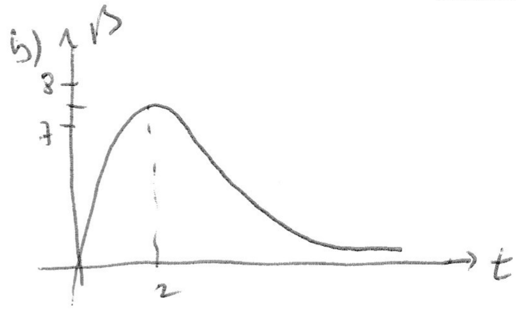
1. $\beta(t) = 10t\cdot e^{-t/2}$

a) $\beta'(t) = 5(2-t)e^{-t/2} = 0$

$\underline{t=2}$

$\underline{\beta(2) = 20/e \approx 7{,}36}$

b)



c) $\beta''(t) = \frac{5}{2}(t-4)e^{-t/2} = 0$

$\underline{t=4}$

d) $r(t) = 20te^{-t/2}$

$R(t) = -40(2+t)e^{-t/2}$ $\qquad R'(t) = -40\left(e^{-t/2} + (2+t)e^{-t/2}\left(-\frac{1}{2}\right)\right)$

$= -40\left(e^{-t/2} - e^{-t/2} - \frac{1}{2}te^{-t/2}\right) = 20te^{-t/2} = r(t)$

$\int_0^8 r(t)\,dt = [R(t)]_0^8 = -40\cdot 10e^{-4} + 40\cdot 2 = 80(e^4-5)e^{-4} \approx \underline{72{,}7 \text{ mg}}$

e) $\beta^* = \beta(t) + \beta(t-4)$

$\beta^{*\prime} = \beta'(t) + \beta'(t-4) = 5(2-t)e^{-t/2} + 5\underbrace{(2-(t-4))}_{(2-t+4)}\underbrace{e^{-\frac{t-4}{2}}}_{e^{-\frac{t}{2}}e^2}$

= ~~[crossed out]~~

$= 5\left(2-t + (2-t)e^2 + 4e^2\right)\underbrace{e^{-t/2}}_{\neq 0} = 0$

$\underline{t = \frac{2+6e^2}{1+e^2} \approx 5{,}52}$ $\qquad \underline{\beta^*(5{,}52) = 10{,}6}$

2. a) $z_1 = 3+i \quad z_4 = 9+4i$

$z_4 = z_1 + 3d \Rightarrow \underline{d} = \frac{z_4 - z_1}{3} = \frac{6+3i}{3} = \underline{2+i}$

$z_{100} = z_1 + 99d = 3+i+198+99i = 201+100i$

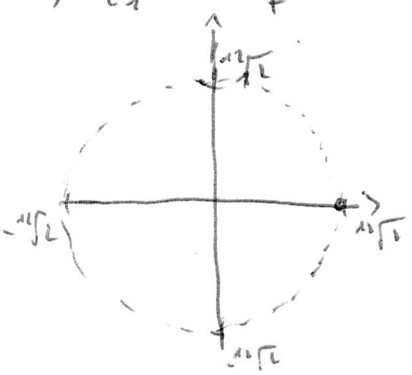
$\underline{\sum_{i=1}^{100} z_i} = \frac{z_1+z_{100}}{2}\cdot 100 = \underline{10200+5050i}$

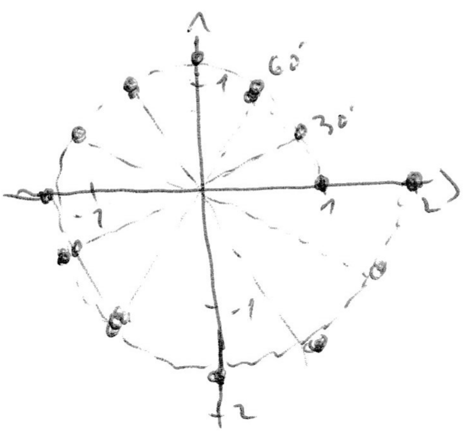
b) $z_1 = 18+6i \quad z_2 = 3+9i \qquad z_2 = q\cdot z_1 \Rightarrow q = \frac{z_2}{z_1} = \frac{3+9i}{18+6i} = \frac{1+3i}{6+2i}$

$\sum_{k=1}^{\infty} z_k = \sum_{k=1}^{\infty} z_1 q^{k-1} = z_1 \sum_{k=1}^{\infty} q^{k-1} = z_1 \sum_{k=0}^{\infty} q^k = z_1\cdot\frac{1}{1-q} = (18+6i)\cdot\frac{1}{1-\frac{1+3i}{6+2i}}$

$= 6(3+i)\cdot\frac{6+2i}{5-i} = \underline{\frac{204}{13} + \frac{228}{13}i}$

c) $z_1 = 1 \quad q = \sqrt[12]{2}\,\text{cis}(30°) = \sqrt[12]{2}\cdot(\cos 30° + i\sin 30°) = \underline{\sqrt[12]{2}\left(\frac{1}{2} + i\frac{1}{2}\sqrt{3}\right) = \frac{1}{2}\sqrt[12]{2} + i\frac{1}{2}\sqrt[12]{2}\sqrt{3}}$





$|z_4| = (\sqrt[12]{2})^3 = 1{,}27$

$|z_7| = 〃 \; 〃^6 = 1{,}41$

$|z_{11}| = 〃 \; 〃^{10} = 1{,}78$

$|z_{13}| = 〃 \; 〃^{12} = 2$

ENH20

d) I $e^{ix} = \cos x + i\sin x$

II $e^{-ix} = \cos(-x) + i\sin(-x) = \cos x - i\sin x$

I−II: $e^{ix} - e^{-ix} = 2i\sin x$

$\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$

e) $z_1 = 5+i \quad z_2 = 7+4i \qquad h = \frac{2z_1z_2}{z_1+z_2} = 2$

3. $E: x+2y-2z+12=0 \quad A(2|0|7) \quad B(26|6|25)$

a) $\vec{AB} = \begin{pmatrix}24\\6\\18\end{pmatrix} = 6\begin{pmatrix}4\\1\\3\end{pmatrix} \quad \vec{n} = \begin{pmatrix}1\\2\\-2\end{pmatrix} \quad \frac{\vec{n}\times\vec{AB}}{3} = \frac{6}{3}\begin{pmatrix}1\\2\\-2\end{pmatrix}\times\begin{pmatrix}4\\1\\3\end{pmatrix} = \begin{pmatrix}16\\-22\\-14\end{pmatrix} = \vec{c}$

$|\vec{AB}| = 6\cdot\sqrt{26} \quad |\vec{n}| = 3$

$\vec{r}_{C_{1,2}} = \vec{r}_B \pm \vec{c} = \begin{pmatrix}42\\-16\\11\end{pmatrix} \,\|\, \begin{pmatrix}10\\28\\39\end{pmatrix} \qquad C(42|-16|11) \quad C'(10|28|39)$

b) $\vec{r}_D = \vec{r}_A \mp \vec{c} = \begin{pmatrix}-14\\22\\21\end{pmatrix} \qquad D(-14|22|21)$

c) $A(6|1|10) \quad B(30|7|28) \quad C(14|29|42) \quad D(-10|23|24)$

$\vec{AB} = \begin{pmatrix}24\\6\\18\end{pmatrix} = 6\begin{pmatrix}4\\1\\3\end{pmatrix} \left(\vec{AC} = \begin{pmatrix}8\\28\\32\end{pmatrix}\right) \left(\vec{n} = \frac{\vec{AB}\times\vec{AC}}{6} = \begin{pmatrix}7\\-22\\-2\end{pmatrix} \quad |\vec{n}| = \sqrt{537}\right)$

$A_{ABCD} = |AB|^2 = 936 \qquad \vec{BC} = \begin{pmatrix}-16\\22\\14\end{pmatrix} \qquad \vec{n} = \frac{\vec{AB}\times\vec{BC}}{12} = \begin{pmatrix}-26\\-52\\52\end{pmatrix} = -26\begin{pmatrix}1\\2\\-2\end{pmatrix}$

$V = \frac{1}{3}Ah \Rightarrow h = \frac{3V}{A} = \frac{3\cdot 7488}{936} = 24 \qquad |\vec{n}| = 78$

$S: \quad \vec{r}_A \pm \frac{24}{78}\vec{n} = \begin{pmatrix}-2\\-15\\26\end{pmatrix} \,\|\, \begin{pmatrix}14\\17\\-6\end{pmatrix} \qquad S_1(-2|-15|26) \quad S_2(14|17|-6)$

S: Ebenen parallel zu E_{ABD} durch $S_{1,2}$

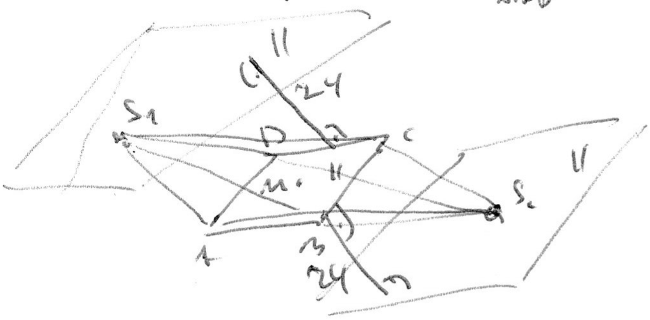
$x+2y-2z+d=0$

$S_1: -2-15\cdot2-2\cdot26+d=0$

$d = +84$

$S_1: x+2y-2z+84=0$

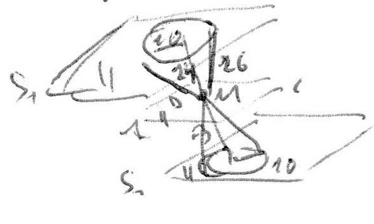
$S_2: x+2y-2z-60=0$



$L = \{ S(x|y|z) \mid x+2y-2z+84=0 \;\vee\; x+2y-2z-60=0 \}$

d) $g: \vec{x} = \frac{\vec{A}+\vec{C}}{2} + s\vec{n} = \begin{pmatrix}10\\15\\26\end{pmatrix} + s\begin{pmatrix}1\\2\\-2\end{pmatrix} \qquad g\cap S_1: (2;-1;42) \quad g\cap S_2: (18;31;10)$

e) Doppelkegel mit Höhe 24 und Radius $10 = \sqrt{26^2-24^2}$; Grundkreise in $S_{1,2}$ Mittelpunkte d)



4 $P(A) = 75\%$ $P(B) = 40\%$ $P(C) = 30\%$ LENH20

1T. a) $P(\text{mind. ein Treffer}) = 1 - P(\text{kein Treffer}) = 1 - 0{,}25 \cdot 0{,}6 \cdot 0{,}7 = 89{,}5\%$

b) $P(B \text{ und } C) = 0{,}4 \cdot 0{,}3 = 12\%$ $P(B \wedge C \mid \text{umfallen}) = \frac{12\%}{89{,}5\%} = 13{,}4\%$

c) $P(\text{mind einen } B \vee C) = 1 - P(\text{kein}) = 1 - 0{,}6 \cdot 0{,}7 = 58\% \to 42\%$ kein Treffer

$P(\text{in } n \text{ Versuchen mind. ein T.}) > 95\%$

$1 - P(\text{kein T. in } n \text{ V.}) > 0{,}95$

$0{,}42^n < 0{,}01$

$n > \log_{0{,}42} 0{,}01 = 5{,}3$ mind. 6 Versuche

2T. Osten: p $1-p$: Norden

a) $A(0|0) \to B(700|300)$: $7 \times$ Ost ; $3 \times$ Nord ; 10 Schritte

$P(A \to B) = p^7 \cdot (1-p)^3 \binom{10}{7}$

b) $P' = 7p^6(1-p)^3 + p^7 \cdot 3(1-p)^2(-1) = \underbrace{p^6(1-p)^2}_{p=0 \quad p=1} \underbrace{(7(1-p) - 3p)}_{p=0{,}7} = 0$

$p = 0$: Min

5. a) $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$

a1) $\vec{a} \times (\vec{a} \times \vec{b}) = -(\vec{a} \times \vec{b}) \times \vec{a} = (\vec{b} \times \vec{a}) \times \vec{a}$ ✓

a2) $\underbrace{(\vec{a} \times \vec{a})}_{\vec{0}} \times \vec{b} = \vec{0} \neq \underbrace{\vec{a} \times (\vec{a} \times \vec{b})}_{\neq 0 \text{ i.d.R.}}$

b) $1{,}11 \cdot 1{,}34 \cdot 1{,}49 = q^3 \to q = 0{,}767$

c) $V_1 = \pi \int_1^9 (y_1^2 - y_2^2)\,dx = \pi \int_1^9 (x - (x-1))\,dx = 8\pi$

$V_2 = \pi \int_0^1 y_1^2\,dx = \pi \int_0^1 x\,dx = \pi \left[\frac{x^2}{2}\right]_0^1 = \frac{\pi}{2}$

$\Big\} \frac{17}{2}\pi$

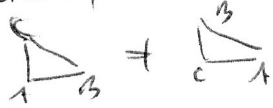
d) $f(x) = ax^4 + bx^2 + c$ Achsensymmetrie: nur gerade E.

$f(2) = 1 \wedge f'(2) = \frac{1}{2} \wedge f''(2) = 0$

$16a + 4b + c = 1 \wedge 32a + 4b = \frac{1}{2} \wedge 48a + 2b = 0$

$a = -\frac{1}{128}$ $b = \frac{3}{16}$ $c = \frac{3}{8}$ $f(x) = -\frac{1}{128}x^4 + \frac{3}{16}x^2 + \frac{3}{8}$

e) 14 Punkte : $14 \cdot 13 \cdot 12 = 2184$ Möglichkeiten, wenn Ecken nummeriert, also



$\frac{14 \cdot 13 \cdot 12}{3!} = 364$ Mgl. falls nicht.