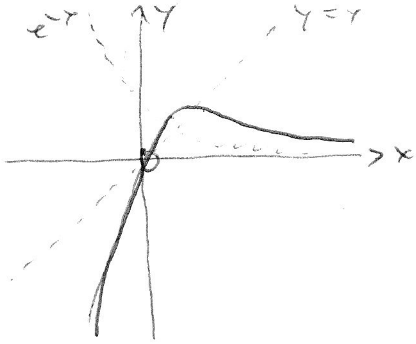
ENH21

1. $f_a(x) = a^2 x e^{-ax}$

a) $f_1(x) = x \cdot e^{-x}$



b) $f_a'(x) = a^2 \underbrace{(1-ax)}_{=0} \underbrace{e^{-ax}}_{\neq 0} = 0$

$\underline{x = \frac{1}{a}} \quad y = a e^{-1}$

$\underline{H_a(\frac{1}{a} | a \cdot e^{-1})}$

c) $x = \frac{1}{a} \longrightarrow a = \frac{1}{x}$

$y = a \cdot e^{-1}$

$\underline{y = \frac{1}{ex}}$

d) $a=1$ $\quad F_1(x) = k \cdot e^{-2x}(2x^2+2x+1)$

$F_1'(x) = \underbrace{-4k}_{=1 \Rightarrow k = -\frac{1}{4}} x^2 e^{-2x} = f_1^2(x) = x^2 e^{-2x}$

$V = \lim\limits_{c \to \infty} \pi \int_0^c f_1^2(x)\,dx = \pi \cdot \lim\limits_{c \to \infty} \left[-\frac{1}{4} e^{-2x}(2x^2+2x+1)\right]_0^c$

$= -\frac{\pi}{4} \lim\limits_{c \to \infty} \Big(\underbrace{\underbrace{e^{-2c}}_{\to 0} \underbrace{(2c^2+2c+1)}_{\to \infty}}_{0 \text{ Exp. stärker als Pot}} - 1\Big) = \underline{\frac{\pi}{4}}$

e) $A = 2 \cdot \lim\limits_{c \to \infty} \int_0^c f_1(x)\,dx = 2 \lim\limits_{c \to \infty} \int_0^c \underset{v}{x} \underset{u'}{e^{-x}}\,dx$

$= 2 \lim\limits_{c \to \infty} \left(-xe^{-x}\Big|_0^c + \int_0^c e^{-x}\,dx\right) = 2 \lim\limits_{c \to \infty} \left(-xe^{-x}\Big|_0^c - [e^{-x}]_0^c\right)$

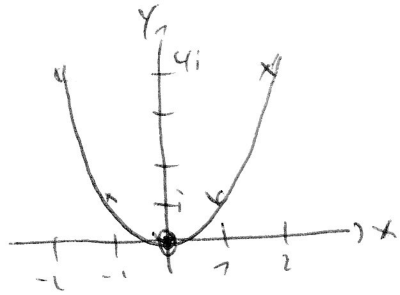
$= 2 \lim\limits_{c \to \infty} (0 - \underbrace{e^{-c}}_{\to 0} + 1) = 2$

CENITC1

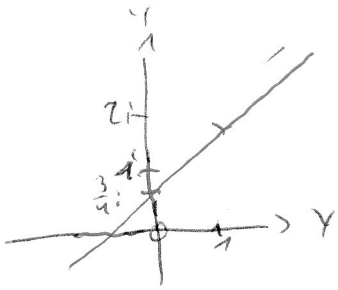
2. $A = \{z \mid Im(z) = (Re(z))^2\}$

$B = \{z \mid 4Re(z) = 4Im(z) - 3\}$

a) A:



B:



b) B: $Im(z) = Re(z) + \frac{3}{4} = Re(z)^2$: A $\quad Re(z) = u$

$u^2 - u - \frac{3}{4} = 0$

$u_1 = -\frac{1}{2} = Re(z) \quad Im(z) = \frac{1}{4} \quad \underline{z_1 = -\frac{1}{2} + \frac{1}{4}i}$

$u_2 = \frac{3}{2} = Re(z) \quad Im(z) = \frac{9}{4} \quad \underline{z_2 = \frac{3}{2} + \frac{9}{4}i}$ $\Big\}$ $A \cap B$

c) A: $|z|^2 = 2450 = Re(z)^2 + Im(z)^2 = Re(z)^2 + Re(z)^4 \quad | Re(z) = u$

$u^4 + u^2 - 2450 = 0 \qquad (u^2+50)(u^2-49) = 0$

$\underline{u_{1/2}} = \underline{\pm 7}$

$\underline{z_1 = 7 + 49i}$

$\underline{z_2 = -7 + 49i}$

d) $z_1 = a + ib \qquad z_2 = (a+ib)(\cos\varphi + i\sin\varphi)$

$\underline{z_2 = a\cos\varphi - b\sin\varphi + (a\sin\varphi + b\cos\varphi)i}$

$z = u \cdot v \qquad |z| = |u| \cdot |v| \qquad |z_2| = |z_1| \cdot \underbrace{|\cos\varphi + i\sin\varphi|}_{1\ ?}$ $\Big\}$ $\underline{\underline{|z_2| = |z_1|}}$

$(\cos\varphi + i\sin\varphi)(\cos\varphi - i\sin\varphi) = \cos^2\varphi + \sin^2\varphi = 1$ ✓

e) $\underline{z = (-1+\sqrt{3}i)^8} = (2\cdot(\cos 120° + i\sin 120°))^8 = 2^8\cdot(\cos 120° + i\sin 120°)^8 \qquad 120°\cdot 8 = 960°$
$\hat{=} 240°$

$= 256\cdot(\cos 240° + i\sin 240°)$

$= 256\left(-\frac{1}{2} - \frac{1}{2}\sqrt{3}i\right) = \underline{-128(1+\sqrt{3}i)}$

LENH21

3\. a) $\frac{x+2y+2z-12}{3}=0$ $\quad \vec{n}=\begin{pmatrix}1\\2\\2\end{pmatrix}$ $\quad |\vec{n}|=3$

$O(0|0|0)$: $\quad \underline{d=\frac{12}{3}=4}$

b) $z=0 \wedge g: \quad t=-4 \quad \underline{G(1|1|0)}$

$g \wedge E: \quad 13+3t+2(9+2t)+2(4+t)-12=0$

$t=-3 \quad \underline{D(4|3|1)}$

c) $\cos\varphi = \frac{\begin{pmatrix}1\\2\\2\end{pmatrix}\begin{pmatrix}3\\2\\1\end{pmatrix}}{3\cdot\sqrt{14}} = \frac{3+4+2}{3\sqrt{14}} \Rightarrow \underline{\varphi = 74{,}5° = 75°}$

d) $A(20|15|-1)$: $\quad \overrightarrow{X_gA} = \begin{pmatrix}-7+3t\\-6+2t\\5+t\end{pmatrix}$ $\quad \overrightarrow{X_gA}\cdot\begin{pmatrix}3\\2\\1\end{pmatrix}=0$ $\quad \underline{P(19|13|6)}$

$14t-28=0$

$\underline{t=2}$

e) Hauptebenen berühren: $M(R|R|R)$
(alle Koo positiv)

Abstand M zu E : R

$\left|\frac{R+2R+2R-12}{3}\right| = R$

$\frac{7R-12}{3} = \pm R$

$(+)$: $\underline{R_1=3}$

$(-)$: $\underline{R_2=1{,}2}$

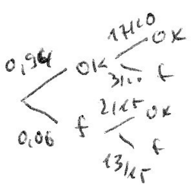
4\. $p=6\%$

a) $P(1 \text{ von } 2 \text{ fehlerhaft}) = 2\cdot p\cdot(1-p) = 2\cdot 0{,}06\cdot 0{,}94 = \underline{11{,}28\%}$

b) $P(3 \text{ von } 50 \text{ ″}) = \binom{50}{3}p^3(1-p)^{47} = \frac{50!}{3!\,47!}\,0{,}06^3\,0{,}94^{47} = \underline{23{,}1\%}$

c) $P(\text{mind. } 10 \text{ v. } 12 \text{ fehlerfrei}) = \binom{12}{10}q^{10}p^2 + \binom{12}{11}q^{11}p + q^{12} = \underline{96{,}8\%}$

d) 2 von 15 f. ✓ | 3 von 200 ok f



$P(ok \mid \checkmark) = \frac{0{,}94\cdot 197/200}{0{,}94\cdot 197/200 + 0{,}06\cdot \frac{2}{15}} = \underline{99{,}6\%}$

e)

140	150	160	170	180	190	200
1	2	3	3	4	2	1

Modus: $\underline{180}$

Median: $\underline{170}$

Mittelwert: $\underline{170{,}625 = \bar{x}}$

$\overline{x^2} = 29368{,}8$

$\sigma^2 = \overline{x^2} - \bar{x}^2 = 255{,}859$

$\underline{\sigma = 16}$

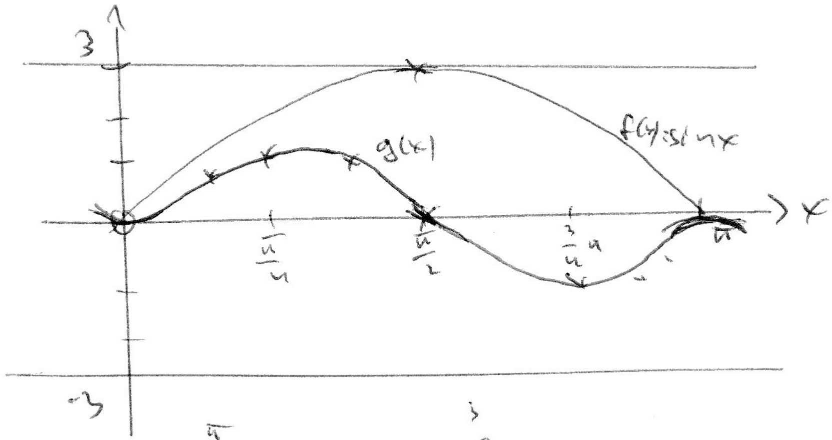
$\underline{\sigma_{n-1} = 17}$

EMHU

5a) $f(x) = \frac{x^2-7x+12}{2x-1} = \underbrace{\frac{1}{2}x - \frac{13}{4}} + \frac{35/4}{2x-1}$

$$y = \frac{1}{2}x - \frac{13}{4}$$

b)



$3\,\underbrace{\sin^2 x}\,\underbrace{\cos x} = 0$

$0/\pi$ doppelt $\quad \frac{\pi}{2} \mid \frac{3}{2}\pi$ einfach

$$A = \int_0^\pi (f-g)\,dx = \int_0^\pi \left(3\sin x - 3(1-\cos^2 x)\cos x\right)dx$$

$$= \int_0^\pi \left(3\sin x - 3\cos x + 3\cos^3 x\right)dx$$

$$= 3\int_0^\pi (\sin x - \cos x)\,dx + \underbrace{3\int_0^\pi \cos^3 x\,dx}_{0}$$

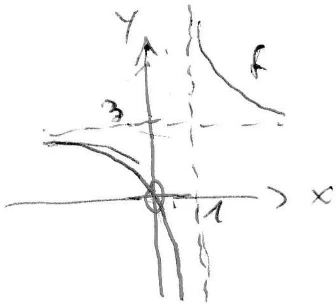
0 weil $\cos x \to \cos^3 x$ punktsymmetrisch in $\frac{\pi}{2}$ und das Intervall ist symmetrisch in $\frac{\pi}{2}$.

$$= 3\left[-\cos x - \sin x\right]_0^\pi$$

$$= -3(-1-1) = \underline{6}$$

c) $\lim\limits_{x\to 3} \frac{e^x - e^3}{5x-15} = \lim\limits_{x\to 3} \frac{e^x}{5} = \underline{\frac{e^3}{5}}$

d) hor. A $y=3$; vert. A $x=1$



$f(x) = 3\frac{x}{x-1}$

e) $\vec{u} = \begin{pmatrix} 1 \\ 2 \\ \ln a \end{pmatrix} \quad \vec{v} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

$$\vec{u} \times \vec{v} = \begin{pmatrix} -\ln a \\ 2\ln a \\ -3 \end{pmatrix}$$

$|\vec{u}\times\vec{v}|^2 = \ln^2 a + 4\ln^2 a + 9 = 34$

$5\ln^2 a = 25$

$\ln a = \pm\sqrt{5}$

$\underline{a_{1,2} = e^{\pm\sqrt{5}}}$