MEN
H22

1. $f_a(x) = x\sqrt{a-x}$; $x \in \mathbb{R}^+_0$

a) $D = \{x \mid 0 \le x \le a\}$

b) $f'(x) = \sqrt{a-x} + x\frac{1}{2\sqrt{a-x}}(-1) = \frac{2a-3x}{2\sqrt{a-x}}$

$\underline{f'(0) = \sqrt{a}}$ $f'(a)$ n.d.

$\lim_{x\to a} f'(a) = -\infty$ senk. Tangente

c) $f'(x) = 0$

$\underline{x = \frac{2}{3}a}$ $\underline{y = \frac{2}{9}\sqrt{3}\,a^{3/2}}$

d) $a = \frac{3}{2}x \rightarrow \underline{y = \frac{1}{2}\sqrt{2}\,x^{3/2}}$

e) $V = \pi \int_0^a f^2(x)\,dx = \frac{4\pi}{3}$

$\int_0^a x^2(a-x)\,dx = \frac{4}{3}$

$-\frac{1}{12}\left[3x^4 - 4ax^3\right]_0^a = \frac{4}{3}$

$\frac{1}{12}a^4 = \frac{4}{3}$

$\underline{a = \pm 2}$

2. a) $\frac{1}{3+4i} = \frac{3-4i}{9-(4i)^2} = \underline{\frac{3}{25} - \frac{4}{25}i}$

b) $z^5 = -32 = -1 \cdot 2^5$ $-1 = (1; 180°)$

$\underbrace{x = 2\cdot\cos\varphi_k \; ; \; y = 2\sin\varphi_k}_{(2\,;\,\varphi_k)}$; $\underline{\varphi_k = 36 + k\cdot 72°}$; $k = 0;1;2;3;4$

c) $z^5 - iz^4 - z + i = 0$ | $\underline{z = i}$ (s)

$(z^5 - iz^4 - z + i) : (z - i) = z^4 - 1$

$z^4 - 1 = \underbrace{(z^2-1)}_{\underline{z=\pm 1}}\underbrace{(z^2+1)}_{\underline{z=\pm i}}$

$1^2 + \sqrt{3}^2 = 4$ d.h. $|1 + i\sqrt{3}| = 2$

d) $z_0 = \frac{1}{2}(\cos 60° + i\sin 60°) = \frac{1}{2}\left(\frac{1}{2} + i\frac{1}{2}\sqrt{3}\right) = \underbrace{\frac{1}{4}(1 + i\sqrt{3})}_{|\,| < 1}$

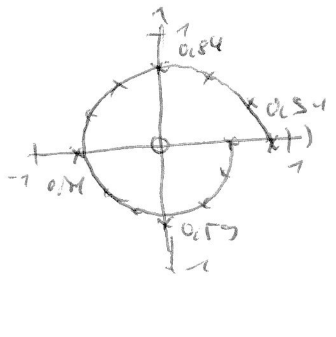
$\sum_{k=0}^{\infty} z_0^k = \underset{|z_0|<1}{\frac{1}{1-z_0}} = \frac{1}{1 - \frac{1}{4}(1+i\sqrt{3})} = \underline{1 + \frac{\sqrt{3}}{3}i}$

e) $t = 0; 6; 12$ $w(t) = 2^{-\frac{t}{12}}(\cos 30° + i\sin 30°)^t = \frac{1}{\sqrt[12]{2}^{\,t}}\left(\cos(t\cdot 30°) + i\sin(t\cdot 30°)\right)$

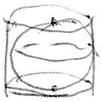
$w(0) = \frac{1}{\sqrt[12]{2}}$ $w(6) = \frac{1}{\sqrt{2}}(-1+0)$ $w(12) = \frac{1}{2}(1\,|\,0)$

$= \left(\frac{1}{\sqrt[12]{2}}\,\middle|\,0°\right)$ $= \left(\frac{1}{\sqrt{2}}\,\middle|\,180°\right)$ $= \left(\frac{1}{2}\,\middle|\,360°\right)$

0,94   0,707



$\boxed{\begin{array}{c}EN\\H22\end{array}}$

3.  $R = 9$ $\quad m: \vec{x} = \begin{pmatrix} 23 \\ -2 \\ -3 \end{pmatrix} + u \begin{pmatrix} -4 \\ 4 \\ -7 \end{pmatrix}$

$t: \vec{x} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + v \begin{pmatrix} 8 \\ 1 \\ -4 \end{pmatrix}$

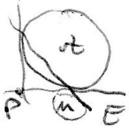
a) $m = t$: $\begin{array}{l} u = 1 \\ v = 2 \end{array}$ $\quad \underline{P(19|2|-10)}$

b) E_T : P ; $\begin{pmatrix} -4 \\ 4 \\ -7 \end{pmatrix} \times \begin{pmatrix} 8 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -9 \\ -72 \\ -36 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 8 \\ 4 \end{pmatrix}$ $\quad d = 5$

$\underline{E_T: \; x + 8y + 4z + 5 = 0}$

c) $\vec{n} \sim \vec{r}$: $\left| \begin{pmatrix} 1 \\ 8 \\ 4 \end{pmatrix} \right| = 9 = r$ $\quad : \quad \vec{r}_M = \vec{r}_P \pm \vec{n} = \underline{\underline{\begin{pmatrix} 20 \\ 10 \\ -6 \end{pmatrix}}} \; \Big| \; \begin{pmatrix} 18 \\ -6 \\ -14 \end{pmatrix}$

d) $\vec{r}_Z = \vec{r}_M \pm \underbrace{\begin{pmatrix} -4 \\ 4 \\ -7 \end{pmatrix}}_{|\,| = 9 = r} = \begin{pmatrix} 16 \\ 14 \\ -13 \end{pmatrix} \; \Big| \; \underline{\underline{\begin{pmatrix} 24 \\ 6 \\ 1 \end{pmatrix}}}$

e) 

$\left. \begin{array}{l} \vec{n} \sim \vec{PZ} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} \\ M \in E \end{array} \right\} d = -154$ $\quad ; \quad \underline{E: \; 5x + 4y + z - 154 = 0}$

4. a) $\left(\frac{1}{3}\right)^5 = \frac{1}{243} = 0{,}411\,\%$

b) $\binom{9}{3} \cdot \left(\frac{1}{3}\right)^3 \cdot \left(\frac{2}{3}\right)^6 = 27{,}3\,\%$

c) $\frac{9!}{3!\,3!\,3!} \cdot \left(\frac{1}{3}\right)^9 = 8{,}53\,\%$

d) $\left(\frac{2}{3}\right)^9 \cdot 3 \approx 7{,}8\,\%$

e) $1 - \left(\frac{1}{3}\right)^9 \cdot 3 - P_{d)} = \frac{224}{243} = 92{,}2\,\%$

5. a) $\frac{e^x - 0}{x - 0} = e^x \rightarrow x = 1$ $\quad A = \int_0^1 (e^x - ex)\,dx = \left[e^x - e\frac{x^2}{2}\right]_0^1 = \underline{\frac{e}{2}}$

b) $\binom{24}{4} \left(\frac{1}{6}\right)^4 \cdot \left(\frac{5}{6}\right)^{4} = \underline{21{,}4\,\%}$

c) $\vec{a} = \begin{pmatrix} 2 \\ 3 \\ t \end{pmatrix}$ $\quad \vec{y} = \begin{pmatrix} t \\ -5 \\ t \end{pmatrix}$ $\quad \cos\gamma = \frac{\vec{a} \cdot \vec{y}}{a \cdot y} = \frac{2t - 15 + t^2}{\sqrt{13 + t^2}\sqrt{25 + t^2}} = 0$ $\quad \underline{\begin{array}{l} t = -5 \\ t = 3 \end{array}}$

$\lim\limits_{t \to \infty} \frac{t^2 + 2t - 15}{\underbrace{\underbrace{\sqrt{t^2 + 13}}_{\to t} \; \underbrace{\sqrt{t^2 + 25}}_{\to t}}_{t^2}} = 1 = \cos\gamma$

$\underline{\gamma = 0^\circ}$

d) $\lim\limits_{x \to \frac{1}{2}} \frac{e^x - \sqrt{e}}{8x^3 - 1} \overset{l}{=} \lim\limits_{x \to \frac{1}{2}} \frac{e^x}{24x^2} = \underline{\frac{1}{6}\sqrt{e}}$

e) $n^3 + 2n \mid 3$

$n = 1$: $1^3 + 2 = 3 \mid 3$ ✓

$n + 1$: $(n+1)^3 + 2(n+1)$

$= n^3 + 3n^2 + 5n + 3$

$= \underbrace{\underbrace{n^3 + 2n}_{|3 \text{ nach V.}} + \underbrace{3n^2 + 3n + 3}_{|3}}_{|3}$ ✓