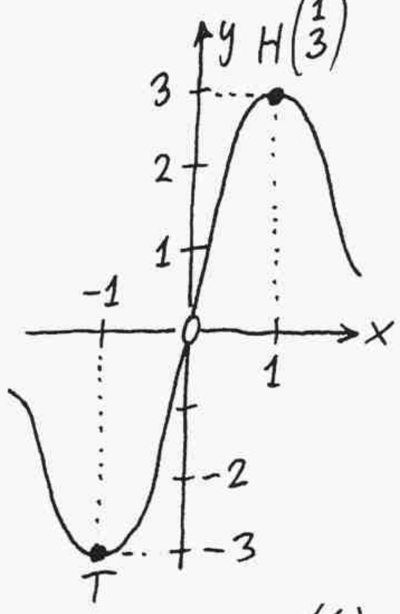
Schweiz. Maturitätsprüfung W2010, Zürich

1 a) Nullstellen: Zähler $= 6x = 0 \rightarrow \underline{\underline{x_1 = 0}}$

Asymptoten: Für $|x| \rightarrow \infty$ schmiegt sich die Funktion an die x-Achse (Funktionsgleichung: $\underline{\underline{y=0}}$)

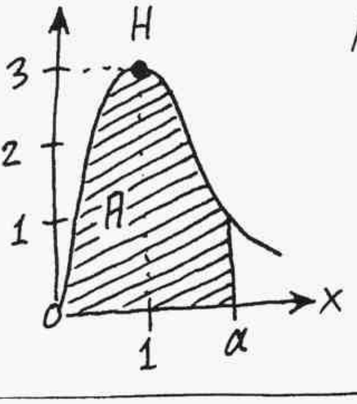


Extrema: $y' = 6\,\dfrac{1-x^2}{(1+x^2)^2}$

$$\left.\begin{array}{l} y' = 0 \rightarrow x^2 = 1 \rightarrow x = \pm 1 \\ f(\pm 1) = \dfrac{\pm 6}{2} = \pm 3 \end{array}\right\} \rightarrow \underline{\underline{T\binom{-1}{-3} \text{ und } H\binom{1}{3}}}$$

b) $\dfrac{d}{dx}\left[3 \cdot \ln(x^2+1)\right] = \dfrac{6x}{x^2+1} = f(x) \rightarrow \underline{\underline{\dfrac{d}{dx}F(x) = f(x)}}$, q.e.d.

c)



$A = F(a) - F(0) = 3\ln(a^2+1) - \cancelto{0}{3\ln 1}$

$= 3\ln(a^2+1) = 9 \rightarrow \ln(a^2+1) = 3 \rightarrow$

$a^2 + 1 = e^3 \rightarrow a = \underline{\underline{+\sqrt{e^3-1}}} = 4.3687$

2.) (a) $m_g = \dfrac{3+6}{12-0} = \dfrac{3}{4}$

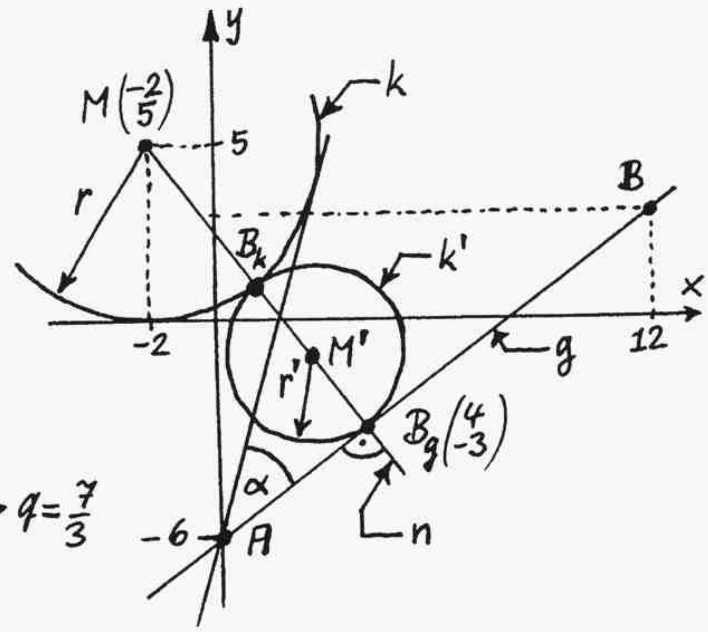
$g: y = \dfrac{3}{4}x - 6$

$m_n = \dfrac{-1}{m_g} = -4/3$

$n: y = -\dfrac{4}{3}x + q$

$M\binom{-2}{5} \in n: 5 = -\dfrac{4}{3} \cdot (-2) + q \rightarrow q = \dfrac{7}{3}$

$n: y = -\dfrac{4}{3}x + \dfrac{7}{3}$



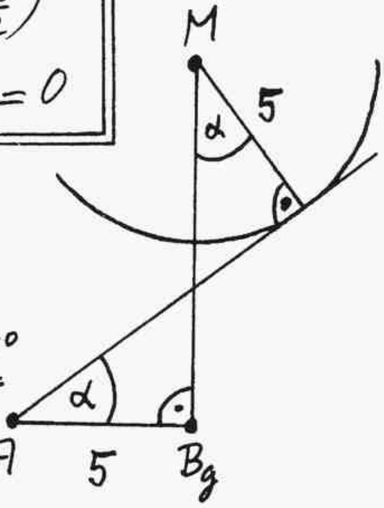
$$n \cap g: \frac{3}{4}x - 6 = -\frac{4}{3}x + \frac{7}{3} \rightarrow \frac{25}{12}x = \frac{25}{3} \rightarrow x = 4,\ y = -\frac{16}{3} + \frac{7}{3} = -3$$

$$B_g\begin{pmatrix}4\\-3\end{pmatrix} \rightarrow \overline{MB_g} = r + 2r' = \sqrt{(4+2)^2 + (-3-5)^2} = 10 \rightarrow r' = \frac{10-r}{2} = \frac{5}{2}$$

$$\vec{r}_{M'} = \vec{r}_{B_g} + \frac{1}{4}\overrightarrow{B_gM} = \begin{pmatrix}4\\-3\end{pmatrix} + \frac{1}{4}\left[\begin{pmatrix}-2\\5\end{pmatrix} - \begin{pmatrix}4\\-3\end{pmatrix}\right] = \begin{pmatrix}5/2\\1\end{pmatrix}$$

$$k': \left(x - \frac{5}{2}\right)^2 + (y+1)^2 = \left(\frac{5}{2}\right)^2$$

$$k': x^2 + y^2 - 5x + 2y + 1 = 0$$

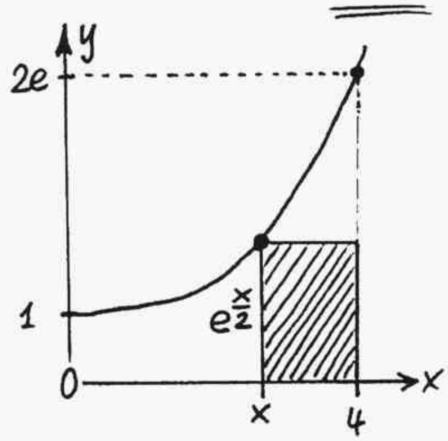


b)

$$\alpha = \arctan\frac{10}{5} - \arctan\frac{5}{10}$$
$$= \arctan 2 - \arctan\frac{1}{2} = \underline{\underline{36.87°}}$$

3\. a) $f \cap g: e^{ax} = 4 \rightarrow x = \frac{\ln 4}{a}$

$$f'(x) = a \cdot e^{ax} \rightarrow f'\left(\frac{\ln 4}{a}\right) = a \cdot e^{\ln 4} = 4a = 1 \rightarrow \underline{\underline{a = \frac{1}{4}}}$$



b) $F = \int_0^4 e^{x/2}\,dx = 2e^{x/2}\Big|_0^4$

$= 2e^2 - 2 = 2(e^2 - 1)$

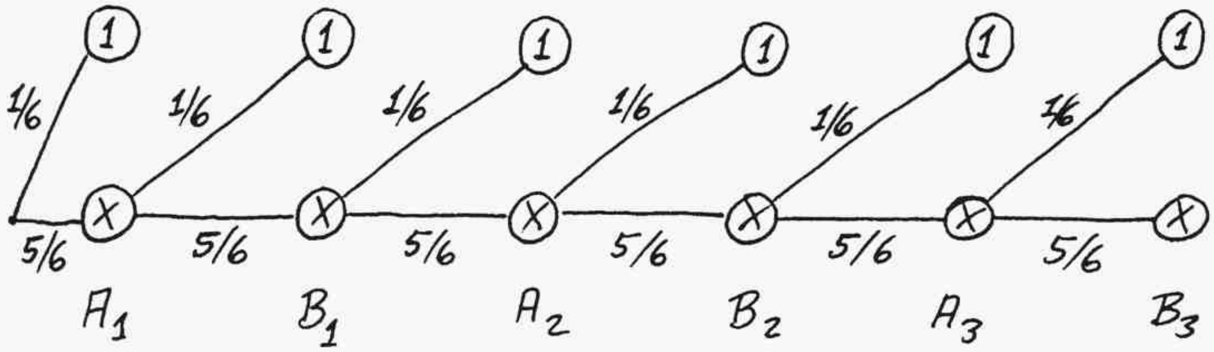
Zielfunktion: $z(x) = (4-x) \cdot e^{x/2}$

Extremum: $\frac{dz}{dx} = \frac{2-x}{2} \cdot \underbrace{e^{x/2}}_{>0} = 0$

$\rightarrow x = 2,\ z(2) = 2e \rightarrow$

$$\frac{z(2)}{F} \cdot 100\% = \frac{2e}{2(e^2-1)} \cdot 100\% = \frac{e \cdot 100\%}{e^2 - 1} = \underline{\underline{42.546\%}}$$

4.)



a) $P_1 = \left(\frac{5}{6}\right)^6 = \underline{\underline{0.3349}}$

b) $P_2 = \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} = \underline{\underline{0.0965}}$

c) $P_3 = \frac{1}{6} + \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} = \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4\right] = \underline{\underline{0.3628}}$

d) $p = \frac{5}{6} \cdot \frac{1}{6} = \frac{5}{36}$, $P_4 = \binom{10}{3} p^3 (1-p)^7 = 120 \cdot \frac{5^3 \cdot 31^7}{36^{10}} = \underline{\underline{0.1129}}$

5. a) $y(0) = 0 \rightarrow d = 0$, $y' = 3x^2 + 2bx + c$, $y'(0) = c = 0$

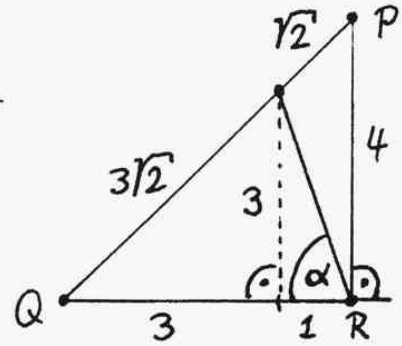
$y(4) = 64 + 16b = 0 \rightarrow b = -4 \rightarrow y = x^3 - 4x^2$,

$y' = 3x^2 - 8x$, $y'' = 6x - 8 = 2(3x - 4) = 0$

$\rightarrow x_w = 4/3 \rightarrow \underline{\underline{W\begin{pmatrix} 4/3 \\ -128/27 \end{pmatrix}, W\begin{pmatrix} 1.333 \\ -4.741 \end{pmatrix}}}$

b1) $\alpha = \arctan 3 = 71.565^\circ$

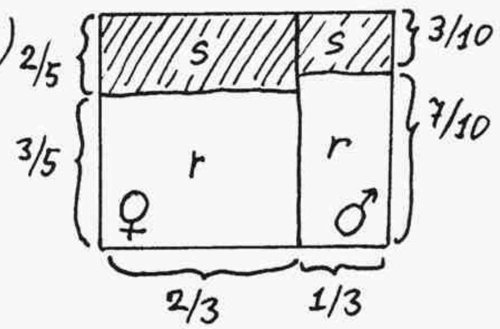
$\frac{\alpha}{90^\circ - \alpha} = \frac{71.565^\circ}{18.435^\circ} = \frac{3.88}{1} \rightarrow \underline{\underline{3.88 : 1}}$



b2) $A_{AQR} = \frac{4 \cdot 3}{2} = 6$, $A_{ARP} = \frac{4 \cdot 1}{2} = 2$

$\rightarrow \underline{\underline{A_{AQR} : A_{ARP} = 3 : 1}}$

c1)



$P(r) = \frac{2}{3} \cdot \frac{3}{5} + \frac{1}{3} \cdot \frac{7}{10} = \underline{\underline{\frac{19}{30}}} = 0.6333$

c2) $P(\text{♀}|s) = \frac{P(\text{♀}\cap s)}{P(s)} = \frac{P(\text{♀}\cap s)}{1-P_r} = \frac{\frac{2}{3}\cdot\frac{2}{5}}{\frac{30}{30}-\frac{19}{30}} = \frac{4}{15}\cdot\frac{30}{11} = \frac{8}{11} = \underline{\underline{0.7273}}$

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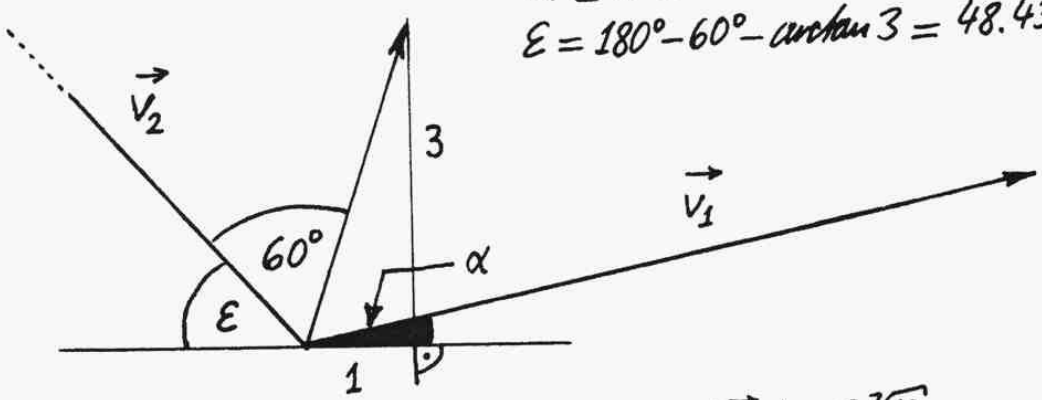
d)

$\alpha = \arctan 3 - 60° = 11.565°$

$\varepsilon = 180° - 60° - \arctan 3 = 48.435°$



$\left|\binom{1}{3}\right| = \sqrt{1^2+3^2} = \sqrt{10} \rightarrow |\vec{v}_1| = |\vec{v}_2| = 2\sqrt{10}$

$$\left.\begin{array}{l} x_1 = 2\sqrt{10}\cos\alpha \\ y_1 = 2\sqrt{10}\sin\alpha \end{array}\right\} \vec{v}_1 = \begin{pmatrix} 6.1962 \\ 1.2679 \end{pmatrix}$$

$$\left.\begin{array}{l} x_2 = -2\sqrt{10}\cos\varepsilon \\ y_2 = 2\sqrt{10}\sin\varepsilon \end{array}\right\} \vec{v}_2 = \begin{pmatrix} -4.1962 \\ 4.7321 \end{pmatrix}$$

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e) $(x^3 + bx - 3) : (x+1) = x^2 - x - 3$

$$\begin{array}{l} x^3 + x^2 \\ \hline -x^2 + bx - 3 \\ -x^2 - x \\ \hline (b+1)x - 3 \quad \leftarrow b+1 = -3 \rightarrow b = -4 \\ -3x - 3 \\ \hline 0 \end{array}$$

$x^2 - x - 3 = 0 \rightarrow x = \frac{1 \pm \sqrt{13}}{2}$

$\rightarrow x_1 = \frac{1+\sqrt{13}}{2} = 2.3028$

$\rightarrow x_2 = \frac{1-\sqrt{13}}{2} = -1.3028$