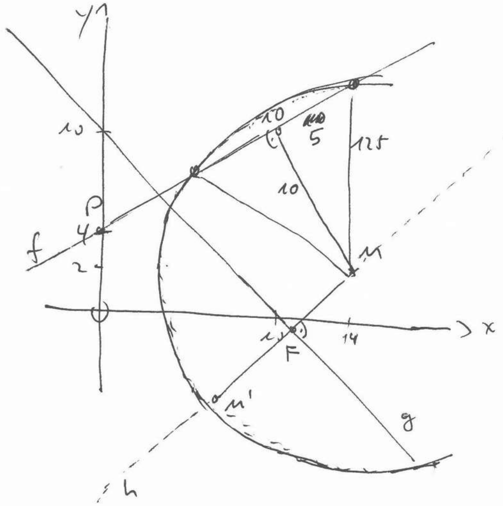
2.



F2011

a)

$g: y = -x + 10$ $\qquad m_1 = -1 \xRightarrow{\perp} m_2 = 1$

$h: y = (x-14)+2 = x - 12$

$g = h$

$\left.\begin{array}{l} x = 11 \\ y = -1 \end{array}\right\} F(11|-1)$

$\overrightarrow{FM'} = \overrightarrow{MF}$

$\vec{M'} - \vec{F} = \overrightarrow{MF}$

$\vec{M'} = \vec{F} + \overrightarrow{MF}$

$= \begin{pmatrix} 11 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -3 \end{pmatrix}$

$\vec{M'} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$ $\qquad \underline{\underline{M'(8|-4)}}$

$\underline{\underline{k': (x-8)^2 + (y+4)^2 = 125}}$

b) Abstand M zu f (siehe oben, Pythagoras) : 10

f durch P : $\quad y = mx + 4$

Hesseform : $\quad \dfrac{mx - y + 4}{\sqrt{m^2+1}} = 0$

Abstand 10 : $\quad \dfrac{mx - y + 4}{\sqrt{m^2+1}} = \pm 10$ $\quad$ für $x = 14$, $y = 2$

$14m - 2 + 4 = \pm 10\sqrt{m^2+1}$

$m_1 = \frac{3}{4} \quad f_1: \underline{y = \frac{3}{4}x + 4}$ $\qquad m_2 = -\frac{4}{3} \quad \underline{y = -\frac{4}{3}x + 4} : f_2$

F2011

3. a) $f(x) = (1-x^2)e^x$

$f'(x) = -2x e^x + (1-x^2)e^x$

$= (1-2x-x^2)e^x$

$f''(x) = (-2-2x)e^x + (1-2x-x^2)e^x$

$= (-1-4x-x^2)e^x$

NST: $\underbrace{(1-x^2)}_{=0} e^x = 0$

$\underline{\underline{x = \pm 1}}$

Ex: $\underbrace{(1-2x-x^2)}_{=0} e^x = 0$

$\underline{x = -1 \pm \sqrt{2}}$ $\quad f''(-1+\sqrt{2}) < 0 \Rightarrow Max(-1+\sqrt{2} \mid 1{,}26)$

$y_1 = -0{,}432$ $\quad f''(-1-\sqrt{2}) > 0 \Rightarrow Min(-1-\sqrt{2} \mid -0{,}43)$

$y_2 = 1{,}255$

WP: $\underbrace{(-1-4x-x^2)}_{=0} e^x = 0$

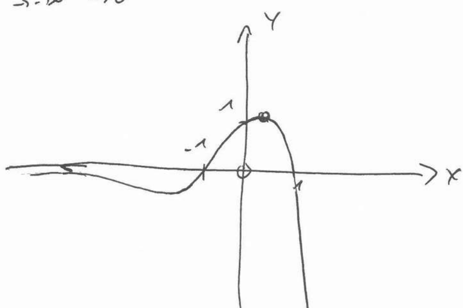
$\underline{x_{1/2} = -2 \pm \sqrt{3}}$. einfache NST, also Vorzeichenwechsel, also WP

$y_1 = 0{,}71$ $\quad WP(-2+\sqrt{3} \mid 0{,}71)$

$y_2 = -0{,}31$ $\quad WP(-2-\sqrt{3} \mid -0{,}31)$

$\lim_{x \to \infty} \underbrace{(1-x^2)}_{\to -\infty} \underbrace{e^x}_{+\infty} = +\infty$

$\lim_{x \to -\infty} \underbrace{(1-x^2)}_{\to -\infty} \underbrace{e^x}_{\to 0} = 0$, da Exp.-Fkt. stärker als Potenzfkt.



F 2011

b) $F(x) = (b + ax - x^2)\, e^x$

$F'(x) = (a - 2x)\, e^x + (b + ax - x^2)\, e^x$

$= (\underbrace{a + b} + \underbrace{(a-2)} x - x^2)\, e^x$

$f(x) = (1 + 0 \cdot x - x^2)\, e^x$

$a + b = 1 \wedge a - 2 = 0$

$b = -1 \leftarrow \underline{a = 2}$

$F(x) = (-1 + 2x - x^2)\, e^x$

$$A = \int_0^1 f(x)\,dx = \left[(-1 + 2x - x^2) e^x\right]_0^1$$

$$= (-1 + 2 - 1)\, e^1 - (-1 + 2 \cdot 0 - 0)\, e^0$$

$$= 0 \quad \underline{\underline{+1}}$$

4. a) P(in 4 Versuchen höchstens einmal die −3)

$= P(\text{keinmal}) + P(\text{einmal}) = \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^3 \cdot \frac{1}{2} \cdot 4 = \underline{\underline{\frac{5}{16} = 31{,}25\,\%}}$

b) P(in 3 Würfen ein neg. Produkt)

$= P(\text{zwei pos., eine neg.}) + P(\text{drei negative})$

$= 3 \cdot \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^3 = \frac{4}{8} = \underline{\underline{\frac{1}{2} = 50\,\%}}$

c) $P(\text{in } n \text{ Versuchen mind. eine } 0) > 0{,}999$

$1 - P(\text{in } n \text{ Versuchen keine } 0) > 0{,}999$

$1 - \left(\frac{5}{6}\right)^n > 0{,}999$

$n > \log_{\frac{5}{6}} 0{,}001 = 37{,}89$

Ab $\underline{\underline{38}}$ Versuchen.

d) P(mehr als drei) = 1 − P(weniger oder gleich 3)

weniger als 3 ist nicht möglich, da 3 Zahlen ermittelt werden müssen.

$= 1 - P(\text{gleich } 3) = 1 - \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{1}{4} = \underline{\underline{\frac{19}{20} = 95\,\%}}$

5. a) $P(3|-2|2)$; $Q(-1|1|6)$; $R(0|0|z)$ | F 2011

$\overrightarrow{PR} \cdot \overrightarrow{QR} = 0$

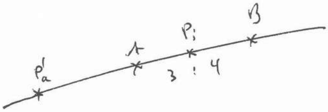
$\begin{pmatrix} -3 \\ 2 \\ z-2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ z-6 \end{pmatrix} = 0$

$-3 - 2 + (z-2)(z-6) = 0$

$z_1 = 1$
$z_2 = 7$

$R_1(0|0|1)$
$R_2(0|0|7)$

b)



$A(1|1)$
$B(15|8)$

P_i : $4 \cdot \overrightarrow{AP_i} = -3\, \overrightarrow{BP_i}$
P_a : $4\, \overrightarrow{AP_a} = 3 \cdot \overrightarrow{BP_a}$

$4 \cdot (\vec{P} - \vec{A}) = \pm 3 (\vec{P} - \vec{B})$

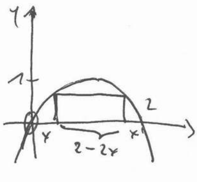
$(4 \mp 3)\vec{P} = 4\vec{A} \pm 3\vec{B}$

$\vec{P} = \dfrac{4\vec{A} \pm 3\vec{B}}{4 \pm 3}$

$\vec{P_i} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$ $\quad$ $P_i(7|4)$

$\vec{P_a} = \begin{pmatrix} -41 \\ -20 \end{pmatrix}$ $\quad$ $P_a(-41|-20)$

c) $f(x) = 2x - x^2 = x(2-x)$



$A = l \cdot b = (2-2x) f(x) = (2-2x)(-x^2+2x)$

$A(x) = 2x^3 - 6x^2 + 4x$ $\quad$ $\mathbb{D} = [0;1]$

$A'(x) = 6x^2 - 12x + 4 = 0$

$x_{1|2} = \dfrac{3 \pm \sqrt{3}}{3}$ $\quad$ $x_1 \notin \mathbb{D}$

$A''(x) = 12x - 12$

$A''\left(\frac{3-\sqrt{3}}{3}\right) < 0 \Rightarrow$ Max

Ränder: $A(0) = 0$, $A(1) = 0$ also Max $x = \dfrac{3-\sqrt{3}}{3}$

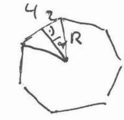
d) Berührung: I. $f(x) = g(x)$
II. $f'(x) = g'(x)$

I. $\ln(ax) = x^2$
II. $\frac{1}{x} = 2x$
$x = \frac{1}{2}\sqrt{2}$ $\quad$ da $x > 0$ ist $x > 0$

in I. $a = \frac{1}{x} e^{x^2}$

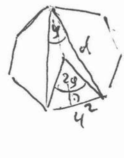
$a = \sqrt{2e}$

e)



$\alpha = \dfrac{360°}{7 \cdot 2}$

$R = \dfrac{2}{\sin\alpha} = 4{,}61$



$2\varphi = \dfrac{360°}{7}$

$\varphi = \dfrac{360}{14}$

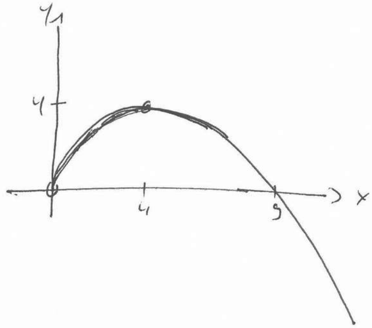
$d = \dfrac{2}{\sin\left(\frac{\varphi}{2}\right)} = 8{,}99$

F2011

1. $f(x) = x(c - \sqrt{x})$; $c > 0$

a) $c = 3$ $\quad f(x) = x(3 - \sqrt{x}) = 0$ $\quad \mathbb{D} = \mathbb{R}_0^+$

$\underline{\underline{x = 0}}$ $\quad \underline{\underline{x = 9}}$



$f(x) = 3x - x^{\frac{3}{2}}$

$f'(x) = 3 - \frac{3}{2} x^{\frac{1}{2}}$

$f''(x) = -\frac{3}{4} x^{-\frac{1}{2}}$

$f'(x) = 0$

$\underline{x = 4}$ $\quad f''(4) < 0 \Rightarrow Max(4|4)$

b) $f(x) = 0$

$\underline{x = c^2}$; $x = 0$

$f(x) = cx - x^{\frac{3}{2}}$

$f'(x) = c - \frac{3}{2} x^{\frac{1}{2}}$

$45°$; d.h. $\tan 45° = \pm 1$ bzw. -1, da Graph fallend.

$f'(c^2) = -1$

$c - \frac{3}{2}\sqrt{c^2} = -1$

$\underline{\underline{c = 2}}$

c) $A = 312{,}5 = \int_0^{c^2} (cx - x^{\frac{3}{2}})\, dx$

$312{,}5 = \left[\frac{c}{2} x^2 - \frac{2}{5} x^{\frac{5}{2}} \right]_0^{c^2}$

$312{,}5 = \frac{1}{2} c^5 - \frac{2}{5} c^5$

$\underline{\underline{c = 5}}$