W13, GF

1. a) $f(x) = x^3 - 3x^2 + 4$

Teiler: $\pm 1, \pm 2, \pm 4$

Lösung: $\underline{x = -1}$

$(x^3 - 3x^2 + 4) : (x+1) = x^2 - 4x + 4$

$\underline{x_{2,3} = 2}$ dopp. -> Extrem

$f'(x) = 3x^2 - 6x$

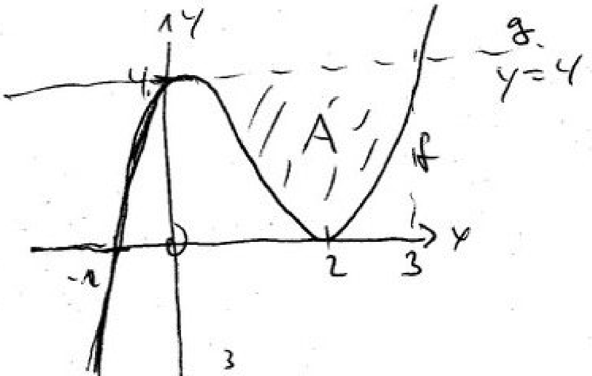
$f'(x) = 0$

$x_1 = 0 \; ; \; x_2 = 2$

$y_1 = 4 \; ; \; y_2 = 0$

$f''(x) = 6x - 6$ $\quad f''(0) = -6 < 0 \Rightarrow \underline{Max\ (0|4)}$

$f''(2) > 0 \Rightarrow \underline{Min\ (2|0)}$



$f = g$

$x^3 - 3x^2 + 4 = 4$

$x_1 = 0$

$\underline{x_2 = 3}$

$$\underline{\underline{A}} = \int_0^3 (g - f)\,dx = \int_0^3 (-x^3 + 3x^2)\,dx = \left[-\frac{x^4}{4} + x^3\right]_0^3 = \underline{\underline{\frac{27}{4}}}$$

b) $f(x) = ax^3 - 3x^2 + 4$

$f'(x) = 3ax^2 - 6x$

$f''(x) = 6ax - 6$

$f''(x) = 0$

$\underline{x = \frac{1}{a}}$ $\quad y = 0 = f(a)$

einfache NST $\quad 0 = 4 - \frac{2}{a^2}$

VZW $\quad \underline{\underline{a = \pm\sqrt{2}/2}}$

immer WP

W13,6F

2. a) $\cos\alpha = \frac{\overline{PQ}\ \overline{PR}}{PQ\ PR} = \frac{\begin{pmatrix}8\\6\end{pmatrix}\begin{pmatrix}-4\\2\end{pmatrix}}{10\ \sqrt{20}} = \frac{-20}{10\sqrt{20}} = -\frac{1}{10}\sqrt{10}$

$\underline{\alpha = 116,57^\circ}$

b) $X(x|0)$ $\quad \vec{XP} \cdot \vec{XQ} = 0$

$\begin{pmatrix}1-x\\1-0\end{pmatrix}\begin{pmatrix}9-x\\7-0\end{pmatrix} = 0$

$(1-x)(9-x) + 7 = 0$

$\underline{x_1 = 2}$

$\underline{x_2 = 8}$

c) $\vec{r}_P = \frac{1}{3}(\vec{r}_R + \vec{r}_Q + \vec{r}_T)$

$\vec{r}_T = 3\vec{r}_P - \vec{r}_R - \vec{r}_Q = \begin{pmatrix}-3\\-7\end{pmatrix}$ $\quad \underline{\underline{T(-3|-7)}}$

d) $M_{PQ} = \begin{pmatrix}5\\4\end{pmatrix}$ $\quad \vec{n}_{PQ} = \begin{pmatrix}-6\\8\end{pmatrix}$ or

$\vec{m}_{PQ} = \begin{pmatrix}5\\4\end{pmatrix} + t\begin{pmatrix}-6\\8\end{pmatrix}$

$M_{PR} = \begin{pmatrix}-1\\2\end{pmatrix}$ $\quad \vec{n}_{PR} = \begin{pmatrix}2\\4\end{pmatrix}$

$\vec{m}_{PR} = \begin{pmatrix}-1\\2\end{pmatrix} + s\begin{pmatrix}2\\4\end{pmatrix}$

$\vec{m}_{PQ} = \vec{m}_{PR}$

$s = \frac{3}{2}$

$t = \frac{1}{2}$ $\quad \underline{\underline{S(2|8)}}$: Umkreismittelpunkt

W13, GF

3. a) $f(x) = x e^x$

$f'(x) = e^x + x e^x = (1+x)e^x$

$f''(x) = e^x + (1+x)e^x = (2+x)e^x$

NST: $\underbrace{x}_{=0}\,\underbrace{e^x}_{\neq 0} = 0$

$x = 0$ einfach

Ext. $\underbrace{(1+x)}\,e^x = 0$

$x = -1$

$y = -\frac{1}{e}$ $\quad f''(-1) > 0 \Rightarrow$ Min$(-1 \mid -\frac{1}{e})$

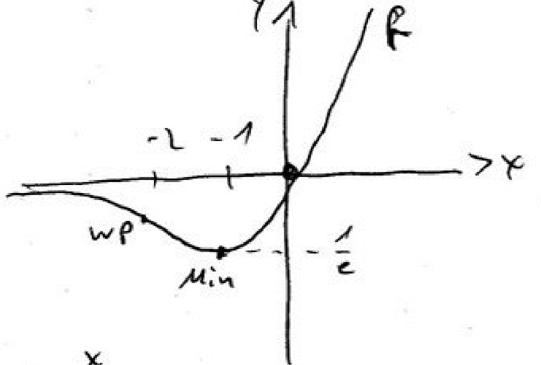
WP $\quad (2+x)e^x$

$x = -2$ einfach, VZW, WP$(-2 \mid -\frac{2}{e^2})$

$y = \frac{-2}{e^2}$

$\lim\limits_{x \to +\infty} \underset{\substack{\downarrow \\ +\infty}}{x}\ \underset{\substack{\downarrow \\ +\infty}}{e^x} = +\infty$

$\lim\limits_{x \to -\infty} \underset{\substack{\downarrow \\ -\infty}}{x}\ \underset{\substack{\downarrow \\ 0}}{e^x} = 0$ (e-Fkt. stärker als Potenz)



b) $F(x) = (x+c)e^x$

$F'(x) = e^x + (x+c)e^x = (\underbrace{1+c}+x)e^x$

$f(x) \overset{!}{=} (0 + x)e^x$

$c = -1$

$F(x) = (-1+x)e^x$

$A = \int_0^1 f(x)\,dx = [F(x)]_0^1 = (-1+1)e^1 - (-1+0)1 = 1$

4. $p_A = 0{,}6$; $p_B = 0{,}7$ $\boxed{W13{,}6F}$

a) $P(\text{in 7 Versuch bei A gen 4 Mal}) = \binom{7}{4} p^4 q^3$

$= \frac{7!}{4!\,3!} 0{,}6^4 \cdot 0{,}4^3 = \underline{29{,}0\,\%}$

b) $P = P(2A, 0B) + P(1A, 1B) + P(0A, 2B)$

$= 0{,}6^2 \cdot 0{,}3^2 + 2 \cdot 0{,}6 \cdot 0{,}4 \cdot 2 \cdot 0{,}7 \cdot 0{,}3 + 0{,}4^2 \cdot 0{,}7^2$

$= \underline{\underline{31{,}2\,\%}}$

c) $P(\text{n Versuchen in A mind.}) \geq 0{,}9999$

$1 - P(\text{0 mal ge}) \geq 0{,}9999$

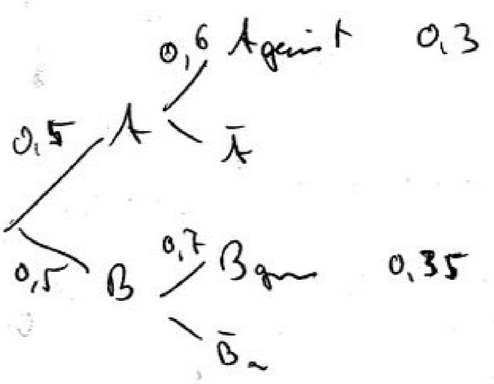
$1 - 0{,}4^n > 0{,}9999$

$0{,}4^n < 0{,}0001$

$n > 10{,}052$

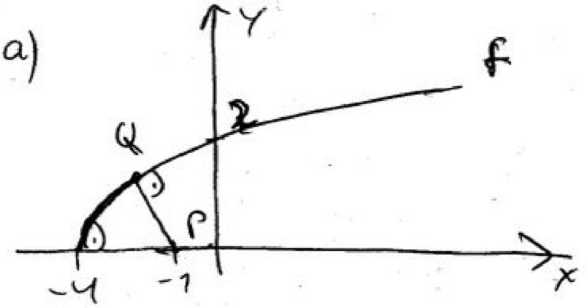
<u>Ab 11 Wiederholungen</u>

d)



$\frac{0{,}3}{0{,}3 + 0{,}35} = \underline{\underline{46{,}2\,\%}}$

5. a)



$m_{PQ} = -\frac{1}{f'}$

$f'(x) = \frac{1}{2\sqrt{x+4}}$

$\frac{f(x) - 0}{x+1} = -2\sqrt{x+4}$

$\sqrt{x+4} = -2\sqrt{x+4}\,(x+1)$

$\underbrace{\sqrt{x+4}}\,(\underbrace{1 + 2(x+1)}) = 0$

$\underline{x = -4}$ $\qquad x = -\frac{3}{2}$

$\underline{y = \frac{1}{2}\sqrt{10}}$ $\qquad Q\left(-\frac{3}{2} \,\middle|\, \frac{1}{2}\sqrt{10}\right)$

b) I $a \ln x = -0{,}5x$ $\quad | \quad f = g$

II $\frac{a}{x} = 2$ $\quad | \quad f' = -\frac{1}{g'}$

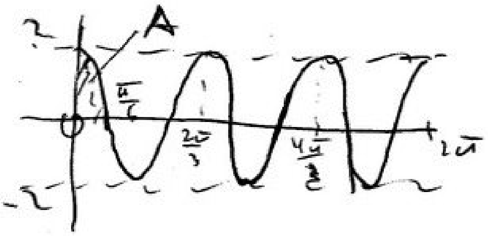
II in I: $2x \cdot \ln x = -\frac{1}{2}x$

$\ln x = -\frac{1}{4}$

$x = e^{-\frac{1}{4}}$

$\underline{\underline{a}} = 2x = \underline{\underline{2e^{-\frac{1}{4}}}}$

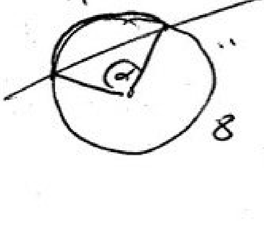
c)



$A = \int_0^{\pi/6} 2\cos(3x)\,dx = \left[\frac{2}{3}\sin(3x)\right]_0^{\frac{\pi}{6}}$

$\underline{\underline{A = 2/3}}$

d)



$\alpha = \frac{1}{9} \cdot 360° = 40°$

$\underline{\underline{A_{Seg}}} = A_S - A_\Delta = \frac{\alpha}{360} \cdot r^2\pi - \frac{1}{2}r^2 \sin\alpha$

$\underline{\underline{= 0{,}44}}$

e) $\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + t\begin{pmatrix} -3 \\ 2 \\ z-3 \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ -13 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad \Big\} \begin{matrix} t = 2 \\ s = -4 \end{matrix}$

$\underline{\underline{z = 4}}$

$\underline{\underline{S(-4 \,|\, 5 \,|\, 5)}}$

$\cos\alpha = \frac{\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}}{\sqrt{14} \cdot 3} \Rightarrow \underline{\underline{\alpha = 108°}}$