

1. $f(x) = x^3 - ax^2, a > 0$

a) $a = 2$

$f(x) = x^3 - 2x^2$

$f'(x) = 3x^2 - 4x$

$f''(x) = 6x - 4$

NST

$x^2(x-2) = 0$

$x=0$ doppelte

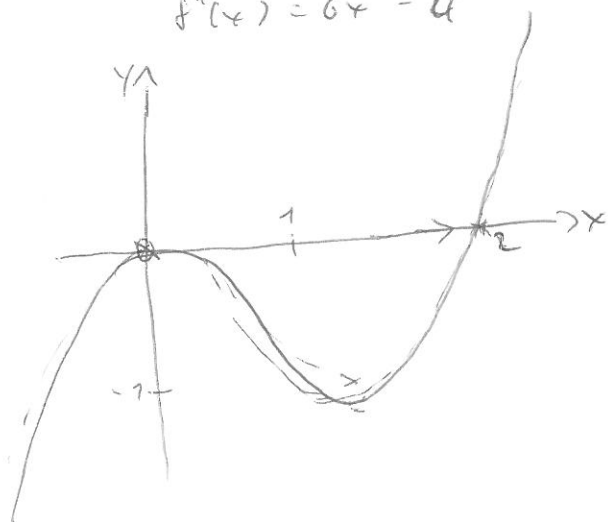
$x=2$ einfach

Ext.

$x(3x-4) = 0$

$x=0 \quad x = \frac{4}{3}$

$y=0 \quad y = -32/27 \approx -1,18$



b) $f(x) = x^3 - ax^2$ NST:

$x^2(x-a) = 0$
 $x=0 \quad x=a$

b1) $A = 108 = - \int f(x) dx$

$-108 = \left[\frac{1}{4}x^4 - \frac{1}{3}ax^3 \right]_0^a$

$-108 = -\frac{1}{12}a^4$

$a=6$

b2) $f'(x) = 3x^2 - 2ax$

$f''(x) = 6x - 2a$

$f''(x) = 0$

$x = \frac{1}{3}a$

$y = f\left(\frac{1}{3}\right) = -\frac{2}{27}a^3$

$y < -2$

$-\frac{2}{27}a^3 < -2$

$a^3 > 27$

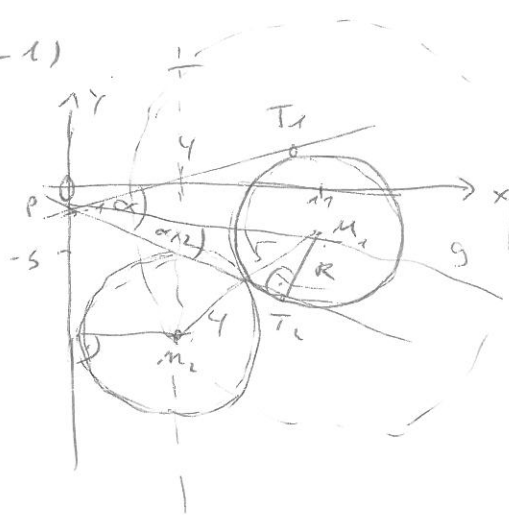
$a > 3$

2. k: $M(11|-3)$ $R=5$ $P(0|-1)$

a) $\sin \frac{\alpha}{2} = \frac{R}{PM} = \frac{5}{\sqrt{11^2+2^2}} = \frac{1}{5}\sqrt{5}$

$\alpha = 53,13^\circ$

$\underline{PT} = \sqrt{PM^2 - R^2}$
 $= \sqrt{125 - 25} = 10$

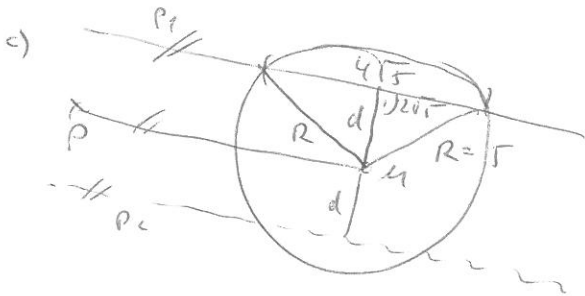


b) $M_1 M_2 = 4+5=9$ $x_{M_2} = 4$

Kreis um M_2 mit Radius 9: $(x-11)^2 + (y+3)^2 = 81$

$x=4$: $y = -3 \pm \sqrt{36} =$
 $y = -3 \pm 4\sqrt{2}$

$M_2(4 | -3 \pm 4\sqrt{2})$



$d = \sqrt{25 - 20} = \sqrt{5}$

PM: $y = -\frac{2}{11}x - 1$

$2x + 11y + 11 = 0$

HNF $\frac{2x + 11y + 11}{\sqrt{125}} = 0$

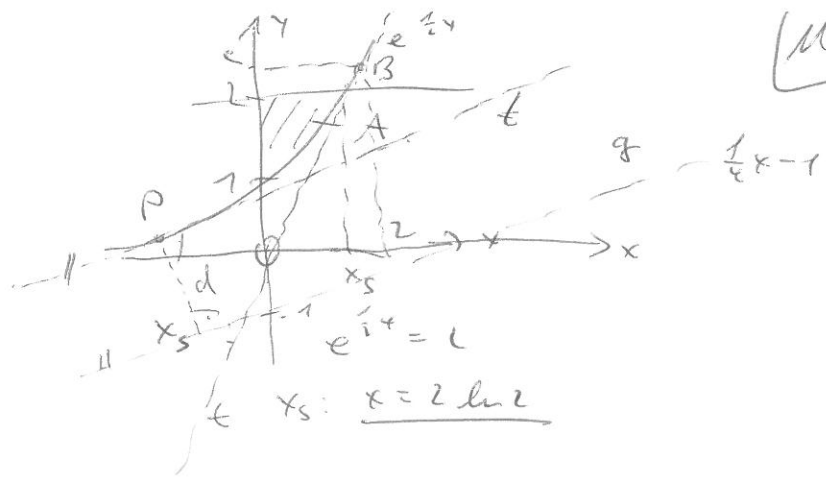
$d = \sqrt{5}$ $\frac{2x + 11y + 11}{5\sqrt{5}} = \pm \sqrt{5}$

$2x + 11y + 11 = \pm 25$

$P_1: 2x + 11y - 14 = 0$

$P_2: 2x + 11y + 36 = 0$

3. $f(x) = e^{\frac{1}{2}x}$



a)

$$A = \int_0^{x_s} (2 - f(x)) dx$$

$$= [2x - 2e^{\frac{1}{2}x}]_0^{2 \ln 2}$$

$$= 4 \ln 2 - (0 - 2e^{\ln 2})$$

A = 4 ln 2 + 4

b) $f'(x) = \frac{1}{2}$

$\frac{1}{2} e^{\frac{1}{2}x} = \frac{1}{4}$

$x = 2 \ln \frac{1}{2} \quad y = \frac{1}{2}$

P(2 ln 1/2 | 1/2)

g: $x - 4y - 4 = 0$
 HNF: $\frac{x - 4y - 4}{\sqrt{17}} = 0$

$$d(P; g) = \left| \frac{2 \ln \frac{1}{2} - 4 \cdot \frac{1}{2} - 4}{\sqrt{17}} \right|$$

$$= \left| \frac{2 \ln \frac{1}{2} - 6}{\sqrt{17}} \right| \approx -1,79$$

c) $\frac{y-0}{x-0} = f'(x)$

$\frac{e^{\frac{1}{2}x}}{x} = \frac{1}{2} e^{\frac{1}{2}x}$

x = 2 y = e B(2|e)

t: y = \frac{e}{2} \cdot x

4, $1 \times 1, 3 \times 3, 5 \times 5, 7 \times 7$ $N = 16$

a) $8 = 1+7 = 7+1$ $4 \cdot \frac{1}{16} \cdot \frac{7}{15} \cdot 2 = \frac{11}{60} = 18,3\%$
 $5+3 = 3+5$ $+ \frac{3}{16} \cdot \frac{5}{15} \cdot 2$

b) $P(\text{in max 3 Züge ein 7})$
 $= P(\text{in erster Zug}) = \frac{7}{16}$
 $+ P(\text{in 2. Zug}) = \frac{9}{16} \cdot \frac{7}{15}$
 $+ P(\text{in 3. Zug}) = \frac{9}{16} \cdot \frac{8}{15} \cdot \frac{7}{14}$

 $\frac{17}{20} = 85\%$

c) $8(5,5, 7,7)$ $\frac{5^2 \cdot 7^2}{16^4}$ Pfad.
 Anzahl Pfade $\binom{4}{2} = \frac{4!}{2! \cdot 2!} = 6$ $\left. \begin{array}{l} \\ \\ \end{array} \right\} 6 \cdot \frac{5^2 \cdot 7^2}{16^4} = 11,2\%$

d) $\bar{x} = \frac{1 \cdot 4 + 3 \cdot 11 + 5 \cdot 15 + 7 \cdot 26}{60} = \frac{157}{30} = 5,23$

$\bar{x}^2 = \frac{1^2 \cdot 4 + 3^2 \cdot 11 + 5^2 \cdot 15 + 7^2 \cdot 26}{60} = \frac{463}{15} = 30,86$

$\sigma^2 = \bar{x}^2 - \bar{x}^2 = \frac{3111}{300}$ $\left(= \frac{(1-\bar{x})^2 \cdot 4 + (3-\bar{x})^2 \cdot 11 + (5-\bar{x})^2 \cdot 15 + (7-\bar{x})^2 \cdot 26}{60} \right)$

$\sigma = \frac{\sqrt{3111}}{30} = 1,87$

a) $P(3 \text{ gewinn}) = 6 \cdot \left(\frac{1}{6}\right)^3 = \frac{1}{36}$

$P(\text{w\u00e4hrt m\u00e4 eine 3 gew}) > 0,99$

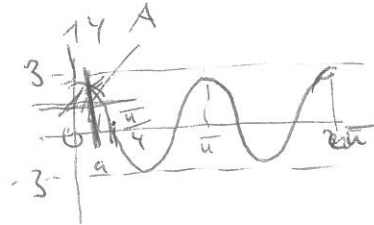
$1 - P(\text{w\u00e4hrt nie 3 gew}) > 0,99$

$1 - \left(\frac{35}{36}\right)^n > 0,99$

$\left(\frac{35}{36}\right)^n < 0,01$

$n > \ln_{\frac{35}{36}} 0,01 = 163,47$

$n > 164$



b) $f(x) = 3 \cos(2x) ; [0; 10]$

$A = \int_0^{\frac{\pi}{4}} f(x) dx = \left[\frac{3}{2} \sin(2x) \right]_0^{\frac{\pi}{4}} = \frac{3}{2}$

$\frac{A}{1} = \frac{3}{4} = \int_0^a f(x) dx = \left[\frac{3}{2} \sin(2x) \right]_0^a = \frac{3}{2} \sin(2a)$

$\sin 2a = \frac{1}{2}$

$2a = \frac{\pi}{6}$

$a = \frac{\pi}{12}$

c) $\vec{AB} = \begin{pmatrix} 8 \\ -6 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} 5 \\ -11 \end{pmatrix} \quad |\vec{AC}| = \sqrt{5^2 + 11^2} = 13$

$\vec{u} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} = \vec{v} \quad |\vec{u}| = 10$

$\vec{v} = \frac{13}{10} \cdot \vec{u} = \frac{13}{10} \begin{pmatrix} 6 \\ 8 \end{pmatrix} = \frac{13}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

($-\vec{v}$ auch \vec{u})

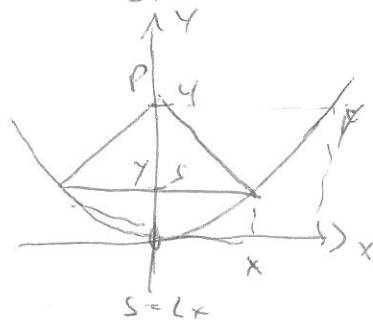
d) $y = \frac{1}{3}x^3 \quad P(0|4)$

$A = \frac{1}{2} g h = \frac{1}{2} \cdot 2x \cdot \left(4 - \frac{1}{3}x^3\right)$

$A = -\frac{1}{3}x^3 + 4x \quad D = [0; 2\sqrt{3}]$

$A' = -x^2 + 4 = 0$

$x = \pm 2$



$\frac{1}{3}x^3 = 4$
 $|x| \leq \sqrt[3]{12} = \sqrt{3}$

$A'' = -2x \quad A''(2) < 0 \Rightarrow \text{Max } x=2 \quad \underline{\underline{A(2) = \frac{16}{3}}}$

Randw: $A(0) = 0$
 $A(2\sqrt{3}) = 0 \quad \Rightarrow$