Mafer 6F
F16

1. $y = x^4 + ax^2 - 4$ A.S.

a) $y = x^4 - 3x^2 - 4 = (x^2+1)(x^2-4)$

NST: $\underline{x = \pm 2}$

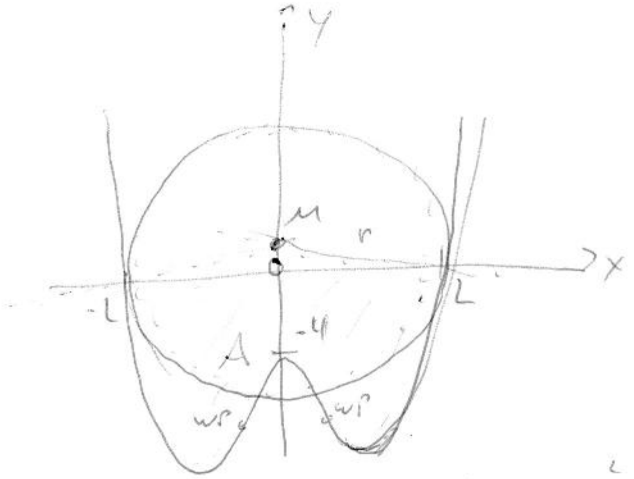
$y' = 4x^3 - 6x = 2x(2x^2 - 3) = 0$

$x = 0 \quad y = -4$

$x = \pm\sqrt{\frac{3}{2}} \quad y = \frac{25}{4}$

$y'' = 12x^2 - 6 \qquad y''(0) = -6 < 0 \Rightarrow Max(0|-4)$

$y''(\pm\sqrt{\frac{3}{2}}) > 0 \Rightarrow Min(\pm\sqrt{\frac{3}{2}}|\frac{25}{4})$



$A = -2\int_0^2 f(x)dx = -2\left[\frac{1}{5}x^5 - x^3 - 4x\right]_0^2 = \underline{\frac{96}{5}}$

Symmetrie, unter x-Achse

$f'(2) = 20 \rightarrow m = -\frac{1}{20}$: r: $y = -\frac{1}{20}(x-2) + 0$

$y = -\frac{1}{20}x + \underline{\frac{1}{10}}$ $\qquad \underline{M(0|\frac{1}{10})}$

b) $y' = 4x^3 + 2ax$

$y'' = 12x^2 + 2a = 0$

$x = \pm\sqrt{-\frac{1}{6}a}$ wg Symmetrie ist y-Wert gleich, also ist Abstand die Diff. x-Wert

$4 = 2\sqrt{-\frac{1}{6}a}$

$\underline{a = -24}$

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2. $\frac{1}{3}$ (5, 6) 2 nach links
$\frac{2}{3}$ (1, 2, 3, 4) 1 nach rechts

G

a) 2 Züge → (C, I)

C : 2 × l $\left(\frac{1}{3}\right)^2$

I : 2 × r $\left(\frac{2}{3}\right)^2$

$\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = \underline{\frac{5}{9}}$

b) 3 Züge → G : 1l 2r
2r 1l
r l r

$3 \cdot \frac{1}{3} \cdot \left(\frac{2}{3}\right)^2 = \underline{\frac{4}{9}}$

c) 5 Würfe → C 3 l 2 r $\binom{5}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 = \underline{\frac{40}{243} = 16{,}4\ \%}$

d) P (in n-Zügen mind. einmal nach links) > 0,999

1 − P(in n Zügen nie) > 0,999

$\left(\frac{2}{3}\right)^n < 0{,}001$

$n > \log_{\frac{2}{3}} 0{,}001 = 17{,}03$

<u>ab 18 Zügen</u>

Mathe 6F
F.06

3. $M(3|2)$ $R=5$

$g: x-2y+6=0$

a) $(x-3)^2+(y-2)^2=5^2$

$(2y-6-3)^2+(y-2)^2=5^2$

$y=2 \quad x=-2 \qquad S_1(-2|2)$

$y=6 \quad x=6 \qquad S_2(6|6)$

$\overrightarrow{MS_1}=\begin{pmatrix}-5\\0\end{pmatrix} \qquad y=\frac{1}{2}x+3$

$\vec{v}=\begin{pmatrix}2\\1\end{pmatrix}$

$\cos\alpha=\frac{\vec{v}\cdot\overrightarrow{MS_1}}{v\cdot MS_1}=\frac{\begin{pmatrix}2\\1\end{pmatrix}\begin{pmatrix}-5\\0\end{pmatrix}}{\sqrt{5}\cdot 5}=\frac{-10}{5\sqrt{5}} \Rightarrow \alpha = 153{,}4°$

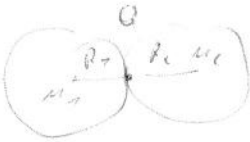
$\alpha_1 = 63{,}4°$

$\overrightarrow{MS_2}=\begin{pmatrix}3\\4\end{pmatrix}$

$\cos\beta=\frac{\vec{v}\cdot\overrightarrow{MS_2}}{v\cdot MS_2}=\frac{\begin{pmatrix}2\\1\end{pmatrix}\begin{pmatrix}3\\4\end{pmatrix}}{\sqrt{5}\cdot 5}=\frac{6+4}{5\sqrt{5}} \Rightarrow \beta = 26{,}6°$

$\alpha_2 = 63{,}4°$

b) $R_2=10 \quad Q(7|-1)$



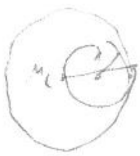
$\overrightarrow{M_1Q}=\begin{pmatrix}4\\-3\end{pmatrix} \qquad M_1Q=5$

$\begin{pmatrix}3\\2\end{pmatrix}+\begin{pmatrix}12\\-9\end{pmatrix}=\begin{pmatrix}15\\-7\end{pmatrix}$

$R_1+R_2=15$

$\overrightarrow{M_1M_2}=3\cdot\overrightarrow{M_1Q}=3\cdot\begin{pmatrix}4\\-3\end{pmatrix}=\begin{pmatrix}12\\-9\end{pmatrix} \qquad M_2(15|-7)$

$\overrightarrow{M_1M_2}=-\overrightarrow{M_1Q}=\begin{pmatrix}-4\\3\end{pmatrix}$



$\begin{pmatrix}3\\2\end{pmatrix}+\begin{pmatrix}-4\\3\end{pmatrix}=\begin{pmatrix}-1\\5\end{pmatrix}$

$M_2(-1|5)$

Mater 6F
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4. $y_1 = \sqrt{x}$

a) $y_2 = 3x^{-2}$

$$\frac{3}{x^2} = \sqrt{x}$$

$$3 = x^{5/2}$$

$$x = 3^{2/5}$$

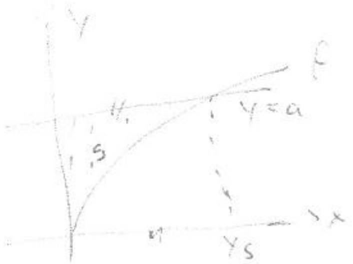
$y_1' = \frac{1}{2\sqrt{x}}$ $\qquad y_1'(3^{2/5}) = \frac{1}{6} 3^{4/5} = m_1$

$y_2' = -6x^{-3}$ $\qquad y_2'(3^{2/5}) = -\frac{2}{3} \cdot 3^{4/5} = m_2$

$$\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = 5{,}64$$

$$\underline{\alpha = 79{,}95^\circ}$$

b)



Y_S: $\quad a = \sqrt{x}$

$$\underline{x_S = a^2}$$

$$\int_0^{a^2} (a - \sqrt{x})\,dx = 9$$

$$\left[ax - \frac{2}{3}x^{\frac{3}{2}}\right]_0^{a^2} = 9$$

$$a^3 - \frac{2}{3}a^3 - 0 = 9$$

$$\underline{a = 3}$$

Mathe GF F26

5, a) 

$g: m = -2 = f'(x)$

$-2 = -3e^{-3x}$

$e^{-3x} = \frac{2}{3}$

$-3x = \ln\frac{2}{3}$

$\underline{x = -\frac{1}{3}\ln\frac{2}{3}} \quad \underline{y = \frac{2}{3}}$

$\underline{P(-\frac{1}{3}\ln\frac{2}{3} \mid \frac{2}{3})}$

b) $\vec{AB}\cdot\vec{AC} = 0$ $\qquad$ $AB = AC$

$\begin{pmatrix}1\\2\\2\end{pmatrix}\begin{pmatrix}x-4\\y-3\\0-2\end{pmatrix} = 0$ $\qquad$ $1^2+2^2+2^2 = (x-4)^2+(y-3)^2+4$

I. $x-4+2(y-3)-4 = 0$ $\qquad$ II $(x-4)^2+(y-3)^2 = 5$

$x = 8-2(y-3)$

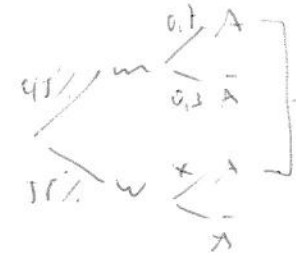
$x = 14-2y$ $\longrightarrow$ $\underline{x_1 = \frac{18}{5} \quad y_1 = \frac{26}{5}}$

$\underline{x_2 = 6 \quad y_2 = 4}$

c) 45% m $\quad$ 55% w

$P(A) = 0{,}4$

$P(\bar{A}\mid m) = 0{,}3$



$0{,}4 = 0{,}45\cdot 0{,}7 + 0{,}55\cdot x$

$\underline{x = 15{,}5\%}$

d) $x^3+2x^2-5x+d = 0$

$\underline{x_1 = 2}$

$\underline{d = -6}$

$(x^3+2x^2-5x-6):(x-2) = x^2+4x+3$

$\underline{x_1 = -3}$

$\underline{x_2 = -1}$

e) $y = \frac{x^2-4x+4}{x-5} = 0$

$x^2-4x+4 = 0$

$(x-2)^2 = 0$

$\underline{x = 2}$ doppelt, also Berührung