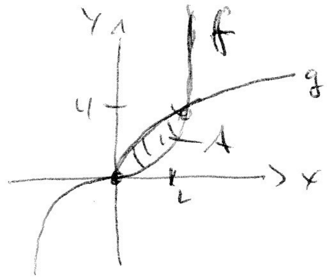
Mathe GF
F22

1. a) $\frac{1}{2}x^3 = \sqrt{8x}$

$\frac{1}{4}x^6 = 8x \quad | x \neq 0$

$x^5 = 32$

$\underline{x = 2}$ Probe ✓




b) $A = \int_0^2 (g-f)\,dx = \left[\frac{2}{3}\sqrt{8}\,x^{\frac{3}{2}} - \frac{1}{8}x^4\right]_0^2 = \frac{10}{3}$

c) i) $d = g(x) - f(x) \to \max$ $\quad d' = \frac{\sqrt{8}}{2}\frac{1}{\sqrt{x}} - \frac{3}{2}x^2 = 0$

$\sqrt{8} = 3x^{5/2}$

$x^5 = \frac{8}{9} \to \underline{x = \sqrt[5]{\frac{8}{9}}}$ $\quad \underline{d\left(\sqrt[5]{\frac{8}{9}}\right) = \frac{5}{9}6^{4/5}}$

ii) $\left.\begin{array}{l} c = \frac{1}{2}x^3 \to x_1 = \sqrt[3]{2c} \\ c = \sqrt{8x} \to x_2 = \frac{c^2}{8} \end{array}\right\} d = \sqrt[3]{2c} - \frac{1}{8}c^2$

$d' = \frac{\sqrt[3]{2}}{3}c^{-2/3} - \frac{1}{4}c = 0$

$c^{5/3} = \frac{4}{3}\sqrt[3]{2} \to \underline{c = \left(\frac{4}{3}\sqrt[3]{2}\right)^{3/5}}$ $\quad \underline{d(c) = \frac{5}{18}6^{4/5}}$

2. $p_M = \frac{1}{6} \quad p_L = \frac{1}{10}$

a) i) W(Lea trifft Dose im vierten V.) $= \bar{p}_M \bar{p}_L \bar{p}_M p_L = \frac{5}{6}\cdot\frac{9}{10}\cdot\frac{5}{6}\cdot\frac{1}{10} = \underline{\frac{1}{16} = 6{,}25\%}$

ii) W(Kein T. in 4 V.) $= (\bar{p}_M \bar{p}_L)^4 = \left(\frac{5}{6}\cdot\frac{9}{10}\right)^4 = \underline{\frac{81}{256} = 31{,}6\%}$

b) W(Mia trifft mind. einmal in N V.) > 99,999%

$1 - W(\text{Mia trifft nie}) > 0{,}99999$

$1 - \left(\frac{5}{6}\right)^N > 0{,}99999$

$N > \log_{\frac{5}{6}} 0{,}00001 = 63{,}14$

Ab 64 Versuchen

c) $\left.\begin{array}{l} W(\text{Mia trifft zuerst}) = (\bar{p}_M \bar{p}_L)^N \cdot p_M \\ W(\text{Lea} \quad\quad\quad ) = (\bar{p}_M \bar{p}_L)^N \cdot \bar{p}_M p_L \end{array}\right\} \frac{(\bar{p}_M \bar{p}_L)^N \bar{p}_M p_L}{(\bar{p}_M \bar{p}_L)^N \bar{p}_M p_L + (\bar{p}_M \bar{p}_L)^N p_M}$

$= \frac{\bar{p}_M p_L}{\bar{p}_M p_L + p_M} = \frac{\frac{5}{6}\frac{1}{10}}{\frac{5}{6}\frac{1}{10} + \frac{1}{6}} = \underline{\frac{1}{3}}$

3. $x^2+y^2=25$

a) $P(-4|3)$ $\vec{OP}=\binom{-4}{3} \xrightarrow{\perp} \vec{t}=\binom{3}{4}$ : $\vec{x}=\binom{-4}{3}+m\binom{3}{4}$

b) $Q(7|1)$ : $7x+y=25$ (Polare)

$x^2+(25-7x)^2=25$

$x_1=3 \quad y_1=4$

$x_2=4 \quad y_2=-3$

$\vec{x}_1=\binom{3}{4}+m\binom{4}{-3}$

$\vec{x}_2=\binom{4}{-3}+m\binom{3}{4}$

c) $\vec{v}=\binom{1}{2}$

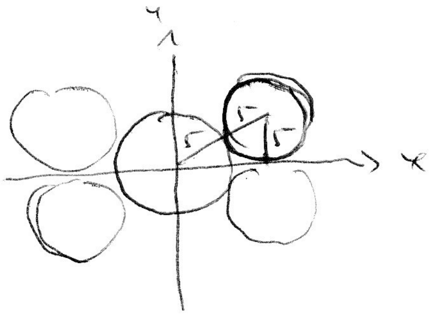
$\vec{n}=\binom{2}{-1}$ $|\vec{n}|=\sqrt{5}$

$\sqrt{5}\cdot\vec{n}=\sqrt{5}\binom{2}{-1}=\binom{2\sqrt{5}}{-\sqrt{5}}$

$\vec{x}_1=\binom{2\sqrt{5}}{-\sqrt{5}}+m\binom{1}{2}$

$\vec{x}_2=\binom{-2\sqrt{5}}{\sqrt{5}}+m\binom{1}{2}$

d)



$\sqrt{10^2-5^2}=\sqrt{75}=5\sqrt{3}$

$(x\pm5\sqrt{3})^2+(y\pm5)^2=25$ (±) alle 4 Kombinationen

4 a) $S_b$: Mitte Seite b -> B $\quad M_b=\frac{\vec{r}_A+\vec{r}_C}{2}=\begin{pmatrix}0\\6\\0\end{pmatrix}$ $\quad \vec{M_bB}=\begin{pmatrix}5\\-6\\0\end{pmatrix}=\vec{S_b}$

$w_\beta$: $\vec{BA}=\begin{pmatrix}-5\\0\\0\end{pmatrix}$ $\quad \vec{BC}=\begin{pmatrix}-5\\12\\0\end{pmatrix}$ $\quad \vec{w}_\beta=\frac{1}{5}\vec{BA}+\frac{1}{13}\vec{BC}=\begin{pmatrix}-18/13\\12/13\\0\end{pmatrix}\sim\begin{pmatrix}-3\\2\\0\end{pmatrix}$

$BC=13$

$\cos\delta=\frac{\begin{pmatrix}\pm5\\\mp6\end{pmatrix}\begin{pmatrix}-3\\2\end{pmatrix}}{\sqrt{61}\ \sqrt{13}}=\frac{27}{\sqrt{61}\sqrt{13}} \rightarrow \delta=16,5°$

$A_{ABC}=\frac{1}{2}\cdot12\cdot5=30$

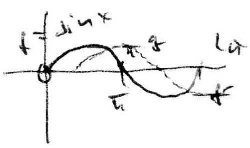
$\vec{BD}=\begin{pmatrix}-5\\0\\z\end{pmatrix}$ $\quad \vec{BD}\times\vec{BC}=\begin{pmatrix}-5\\0\\z\end{pmatrix}\times\begin{pmatrix}-5\\12\\0\end{pmatrix}=\begin{pmatrix}-12z\\5z\\-60\end{pmatrix}$ $\quad \frac{1}{4}(169z^2+60^2)=30^2\cdot4$

$z=\pm\frac{60}{13}\sqrt{13}$

b) $y=\frac{2}{a}x^{-1}-\frac{1}{a}x^{-2}$ $\quad y'=-\frac{2}{a}x^{-2}+\frac{2}{a}x^{-3}$ $\quad y''=+\frac{4}{a}x^{-3}+-\frac{6}{a}x^{-4}$ $\quad y''=0$, $x=3/2$ $\quad y=\frac{8}{9a}=2\cdot\frac{3}{2}$

$a=8/27$

c)



$d=f-g=\sin x-\sin(x-\frac{\pi}{3})$

$d'=\cos x-\cos(x-\frac{\pi}{3})=0$

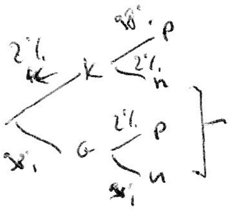
$\cos x-\cos x\cos\frac{\pi}{3}-\sin x\sin\frac{\pi}{3}=0$

$\frac{1}{2}\cos x-\frac{1}{2}\sqrt{3}\sin x=0$

$\tan x=\frac{1}{\sqrt{3}}$ $\quad x=\frac{\pi}{6}+k\cdot\pi$ $\quad x=\frac{7}{6}\pi$

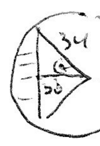
$d=1$

d)



$\frac{98\%\cdot98\%}{98\%\cdot98\%+2\%\cdot2\%}=99,96\%$

e) i) 2·



$\cos\alpha=\frac{30}{34}$

$\alpha=28,07°$

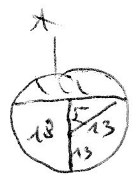
$A=\frac{2\alpha}{360°}\cdot34^2\pi-\frac{1}{2}\cdot34^2\cdot\sin(2\alpha)$

$=566,391-480$

$=86,391$

$2A=172,782$

ii)



$=\frac{5}{13}$

$\alpha=67,38°$

$A=\frac{2\alpha}{360°}\cdot13^2\pi-\frac{1}{2}13^2\sin(2\alpha)$

$=198,745-60=138,75$