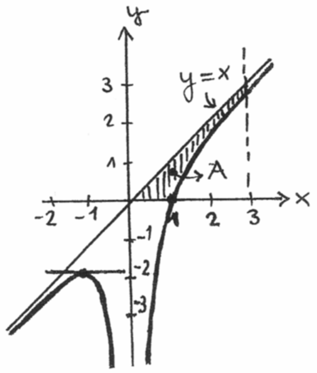
EM Herbst 2001, Neuchâtel : MATHEMATIK ABDE (Doppel-seitig)

Lösungen



1) $f(x) = \frac{x^3-1}{x^2} = x - x^{-2}$

a) $\mathbb{D}_f = \mathbb{R}\setminus\{0\}$ $\quad \underline{x=0}$; Pol 2. Ordn

y-Achse ist vertikale Asymptote

NS: $\left.x^3 = 1\right\}$ $\underline{x=1}$

schiefe Asymptote: $\underline{y=x}$ $\begin{pmatrix}|x|\to\infty \\ x^{-2}\to 0\end{pmatrix}$

$f'(x) = 1 + 2x^{-3}$

$f'(x) = 0 \quad \left. x^3 = -2 \right\}$ $\quad x = -\sqrt[3]{2} \; (-1{,}26)$

$y = -3/\sqrt[3]{4} \; (-1{,}89)$

$\underbrace{}_{\text{rel. Max. } (f''(-\sqrt[3]{2}) = -6/\sqrt[3]{16})}$

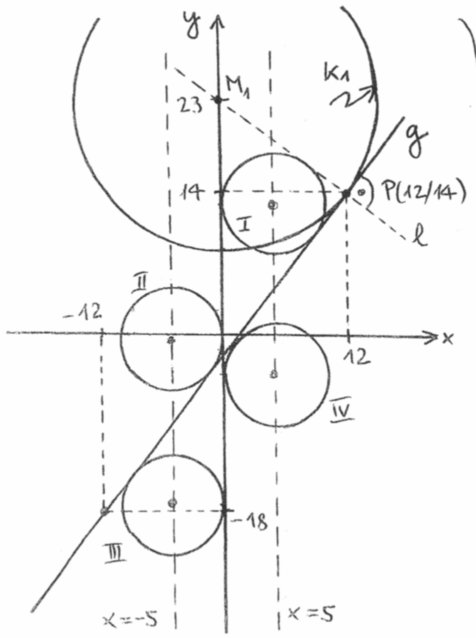
$f''(x) = -6x^{-4}$

$\underbrace{}_{\text{keine WP ; } (f''(x)\neq 0)}$

b) $A_b = -\int_1^b f(x)\,dx + b^2/2 = -\left[\frac{x^2}{2} + x^{-1}\right]_1^b + \frac{b^2}{2} \; \underline{= -\frac{1}{b} + \frac{3}{2}}$

$b = 3$; $\underline{A_3 = \frac{7}{6}}$ $\quad \left.\begin{matrix} b\to\infty \\ \frac{1}{b}\to 0\end{matrix}\right\}$ $\underline{A_\infty = \frac{3}{2}}$

2)



a) $g: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix} + t\begin{pmatrix}3\\4\end{pmatrix}$ $\left.\right\}$ $\left. x_1 = 12 \right\}$ $t = 3$, $y_1 = 14$

$\underline{P(12/14)}$

Lot auf g durch P:

$l: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}12\\14\end{pmatrix} + s\begin{pmatrix}-4\\3\end{pmatrix}$ $\left.\right\}$ $\left. x = 0 \right\}$ $s = 3$, $y = 23$

$|\overrightarrow{PM_1}| = \underline{R_1} = \left|\begin{pmatrix}-12\\9\end{pmatrix}\right| \underline{= 15}$ $\qquad \underline{M_1(0/23)}$

$K_1: \; (x-0)^2 + (y-23)^2 = 15^2$

$\underline{\underline{x^2 + (y-23)^2 = 225}}$

b) $g: \left.\begin{matrix} x = 3+3t \\ y = 2+4t \end{matrix}\right\}$ Koord. form: $4x - 3y - 6 = 0$

Für alle vier Kreismittelpunkte:

Hesse-Form: $\frac{4x-3y-6}{5} = \pm 5$

$\left\{\begin{matrix} \left. x = +5 \right\} \; y = -\frac{11}{3} \; ; \; +13 \\ \left. x = -5 \right\} \; y = -17 \; ; \; -\frac{1}{3} \end{matrix}\right.$

Lösungskreise: I $(x-5)^2 + (y-13)^2 = 25$

II $(x+5)^2 + (y+\frac{1}{3})^2 = 25$

III $(x+5)^2 + (y+17)^2 = 25$

IV $(x-5)^2 + (y+\frac{11}{3})^2 = 25$

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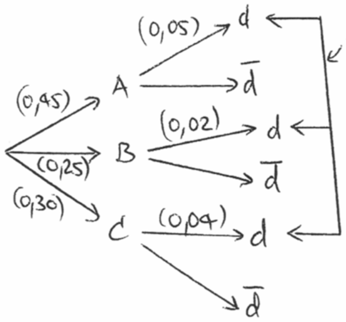
3)



a) $P = 0{,}45\cdot 0{,}05 + 0{,}25\cdot 0{,}02 + 0{,}30\cdot 0{,}04 = \underline{\underline{0{,}0395}}$

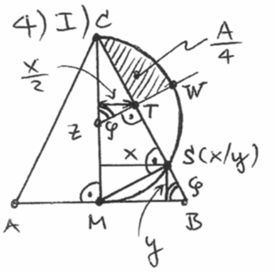
b) $P(C\backslash d) = \frac{P(C\cap d)}{P_d} = \frac{0{,}30\cdot 0{,}04}{0{,}0395} = \underline{\underline{0{,}3038}}$

c) $P(A\backslash \bar d) = \frac{P(A\cap \bar d)}{P(\bar d)} = \frac{0{,}45\cdot 0{,}95}{(1-0{,}0395)} = \underline{\underline{0{,}4451}}$

d) $\overline{V}(26,8) = 26^8 = 2{,}088\cdot 10^{11}$

e) $V(26,6) = \frac{26!}{20!} = 165'765'600$

4) I)



b) Ähnlichkeit: $\underbrace{\frac{\overline{MB}}{\overline{MC}} = \frac{\overline{BS}}{\overline{MS}} = \frac{\overline{MS}}{\overline{CS}} = \frac{1}{2}}_{2\overline{BS} = \frac{\overline{CS}}{2}}$ } $\overline{CS} = 4\overline{BS}$ (S teilt $\overline{BC}$ im Verhältnis 1:4)

$x = \frac{4}{5}$ $y = \frac{2}{5}$

$\overline{AS} = \sqrt{y^2 + (1+x)^2} = 1{,}844$

a) $A = 4\cdot\left(A_{\text{Sektor } ZWC} - A_{\text{Dreieck } ZTC}\right) = 4\cdot\left(\frac{\pi}{360°}\varphi - \frac{1}{2}(1)\frac{2}{5}\right)$; wobei $\sin\varphi = \frac{2}{\sqrt{5}}$; $\varphi = 63{,}435°$

$A = 4\cdot(0{,}5536 - 0{,}2) = 1{,}4143$

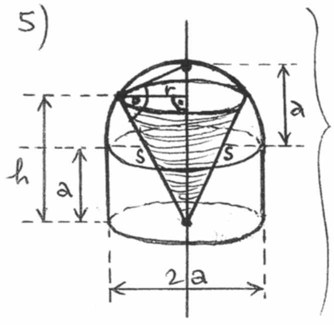
II) $V = \frac{\pi d^3}{6}$; $d = \sqrt[3]{\frac{6V}{\pi}} = 9{,}1416$ cm } (100g Au / $\rho = 19{,}29$ g/cm³)

$V_{Au} = \frac{m}{\rho} = 5{,}184$ cm³

Volumen der vergoldeten Eisenkugel: $V^* = V + V_{Au} = 405{,}184$ cm³

$d^* = \sqrt[3]{\frac{6V^*}{\pi}} = 9{,}1809$ cm } $\Delta d = 0{,}0393$ cm (Durchmesserunterschied) } $D = 0{,}01965$c (Goldschichtdic

5)



$r^2 = h\cdot(2a-h)$; $s^2 = r^2 + h^2 = 2ah$

$M = \pi r s = \pi\sqrt{h(2a-h)}\sqrt{2ah}$

Ist M max, so auch M^2: $M^2(h) = (2a\pi^2)[2ah^2 - h^3]$

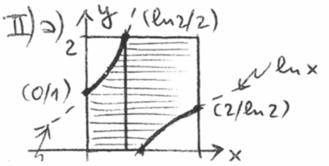
$(M^2(h))' = (2a\pi^2)[4ah - 3h^2] = 0$

h = 0 unmöglich; also $h = \frac{4a}{3}$

6) I) $(1-k)x^2 + x + (3-5k) = 0$ } $D = 0 = 1 - 4(1-k)(3-5k) = -20k^2 + 32k -$

$k_{1,2} = \frac{-32 \pm \sqrt{1024 - 880}}{-40}$ } $k_1 = 0{,}5$; $k_2 = 1{,}1$

II) a)



b) $A_{Ecke} = 2\ln 2 - \int_0^{\ln 2} e^x\,dx$
$= 2\ln 2 - \left[e^x\right]_0^{\ln 2}$
$= 2\ln 2 - [2-1]$

$A_{Rest} = 4 - 2A_{Ecke}$
$= 4 - 4\ln 2 + 2$
$= 6 - \ln 16$
= [illegible]