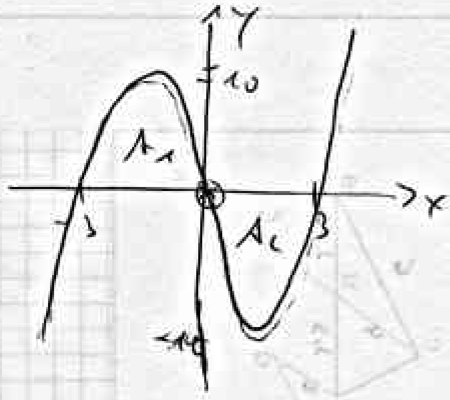
Matur, Sommer 2009



1. $f(x) = x^3 - 9x$

NST: $x = 0, \pm 3$

a)

$f'(x) = 3x^2 - 9$

$f'(0) = -9 = \tan\alpha$

$\underline{\alpha = -83{,}7^\circ = 96{,}3^\circ}$

$f(-x) = f(x)$ Punktsymmetrie, also $\underline{A_1 = A_2}$

$\underline{A_2} = -\int_0^3 f(x)\,dx = -\left[\frac{1}{4}x^4 - \frac{9}{2}x^2\right]_0^3 = \underline{\frac{81}{4}}$

b) $y = -x$ Wendetangente, $m = -1$

Punktsymmetrie: $(0|0)$ ist WP

$f'(x) = 3x^2 - b$

$f'(0) = -b = -1$

$\underline{b = 1}$

c) $f(x) = 0$

$x^3 - bx = 0$

$x(x^2 - b) = 0$

$x = 0, x = \pm\sqrt{b}$

$-\int_0^{\sqrt{b}} f(x)\,dx = 1$

$\left[\frac{1}{4}x^4 - \frac{1}{2}bx^2\right]_0^{\sqrt{b}} = -1$

$-\frac{b^2}{4} = -1$

$\underline{b = \pm 2}$

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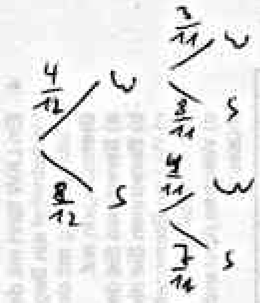
2. a)



1) $P(ww) + P(ss) = \frac{4}{12}\cdot\frac{3}{11} + \frac{8}{12}\cdot\frac{7}{11} = \frac{68}{132} = 51{,}5\%$

2) $P(sw) + P(ws) = 1 - \text{"} = \frac{64}{132} = 48{,}5\%$

b) 2 Kugeln $P(ww)+P(ss) = \frac{68}{132} = 51{,}5\%$

3 Kugeln $P(www)+P(sss) = \frac{4\cdot3\cdot2 + 8\cdot7\cdot6}{12\cdot11\cdot10} = \frac{360}{1320} = 27{,}3\%$

4 Kugeln $P(wwww)+P(ssss) = \frac{4\cdot3\cdot2\cdot1 + 8\cdot7\cdot6\cdot5}{12\cdot11\cdot10\cdot9} = \frac{1704}{11880} = 14{,}3\%$

zu erwartende durchschnittliche Gewinn

2 Kugeln: $\frac{68}{132}\cdot 10 = 5{,}15$

3 ": $\frac{360}{1320}\cdot 20 = 5{,}45$

4 ": $\frac{1704}{11880}\cdot 40 = \underline{5{,}74}$

Bei 4 Kugeln wird der Gewinn maximal (5,74), aber man macht durchschnittlich immer noch 0,26 Verlust.

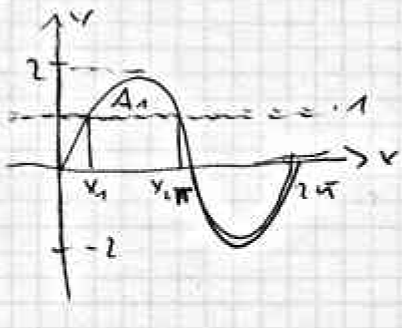
3. a)

$$\left.\begin{array}{lll} \text{I} & \vec{a}\cdot\vec{b} = 0 \to & -3+2x+1=0 \\ \text{II} & \vec{a}\cdot\vec{c} = 0 \to & -1+2y-z=0 \\ \text{III} & \vec{b}\cdot\vec{c} = 0 \to & 3+xy-z=0 \end{array}\right\} \quad \underline{x=1},\ \underline{y=4},\ \underline{z=7}$$

b)

$$\left.\begin{array}{l} lb\,x - lb\,y = 3 \to lb\left(\frac{x}{y}\right) = 3 \to \frac{x}{y} = 2^3 \to x = 8y \\ x - 6y = 6 \to x = 6+6y \longrightarrow x = 6+6y \end{array}\right\} \quad \underline{x=24},\ \underline{y=3}$$

c)



$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x_1 = \frac{\pi}{6}$

$x_2 = \frac{5}{6}\pi$

$$\left.\begin{array}{l} A = \int_0^{\pi} 2\sin x\,dx = 2\left[-\cos x\right]_0^{\pi} = 2 \\ A_1 = \int_{\frac{\pi}{6}}^{\frac{5}{6}\pi} (2\sin x - 1)\,dx = \left[-2\cos x - x\right]_{\frac{\pi}{6}}^{\frac{5}{6}\pi} = 2\sqrt{3} - \frac{2}{3}\pi \end{array}\right\} \quad \underline{\frac{A_1}{A} = \sqrt{3} - \frac{\pi}{3} = 68{,}5\%}$$

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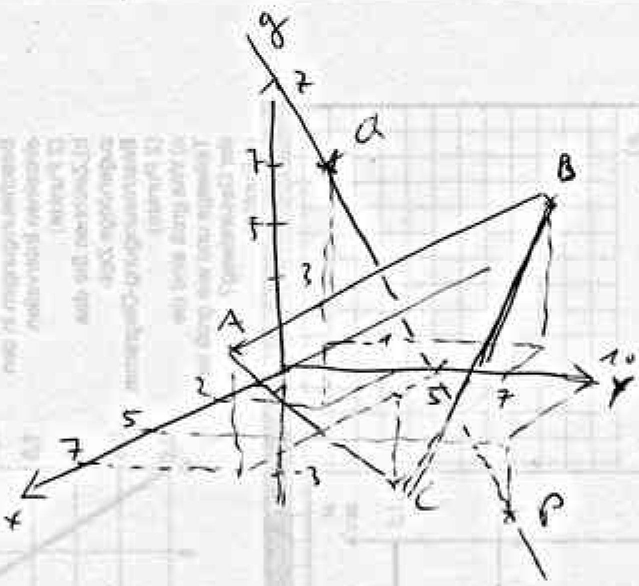
4 a)



b) $\vec{AB} = \begin{pmatrix}-8\\2\\2\end{pmatrix}$ $\quad \vec{AC} = \begin{pmatrix}-6\\0\\-6\end{pmatrix}$; $\vec{BC} = \begin{pmatrix}2\\-2\\-8\end{pmatrix}$

$\vec{AB}, \vec{BC}$: gleiche Komponenten (Betrag): gleichlang

$$\left.\begin{array}{l} |\vec{AB}| = \sqrt{72} = |\vec{BC}| \\ |\vec{AC}| = \sqrt{72} \end{array}\right\} \underline{\text{gleichseitig}} \Rightarrow \underline{\alpha = \beta = \gamma = 60^\circ}$$

c) $g: \vec{x} = \begin{pmatrix}2\\4\\7\end{pmatrix} + \lambda \vec{QP} = \begin{pmatrix}2\\4\\7\end{pmatrix} + \lambda \begin{pmatrix}-3\\-6\\9\end{pmatrix}$

$g \cap (xy)$: $z = 0 = 7 + 9\lambda$

$\lambda = -\frac{7}{9} \rightarrow x = 2 \mp 3\cdot\left(-\frac{7}{9}\right) = +13/3$

$y = 4 - 6\left(-\frac{7}{9}\right) = 26/3$ $\qquad \underline{X\left(+\frac{13}{3} \,\middle|\, \frac{26}{3} \,\middle|\, 0\right)}$

d) $h(B,C): \vec{x} = \begin{pmatrix}-1\\7\\5\end{pmatrix} + \mu \begin{pmatrix}2\\-2\\-8\end{pmatrix}$

$h \cap g \quad \begin{pmatrix}-1\\7\\5\end{pmatrix} + \mu \begin{pmatrix}2\\-2\\-8\end{pmatrix} = \begin{pmatrix}2\\4\\7\end{pmatrix} + \lambda \begin{pmatrix}-3\\-6\\9\end{pmatrix}$ falsch

$h(A,B): \vec{x} = \begin{pmatrix}7\\5\\3\end{pmatrix} + \mu \begin{pmatrix}-8\\2\\2\end{pmatrix}$

$h \cap g \quad \begin{pmatrix}7\\5\\3\end{pmatrix} + \mu \begin{pmatrix}-8\\2\\2\end{pmatrix} = \begin{pmatrix}2\\4\\7\end{pmatrix} + \lambda \begin{pmatrix}+3\\+6\\-9\end{pmatrix}$

$\lambda = 1/3$ $\qquad S(3|6|4)$

$\mu = 1/2$

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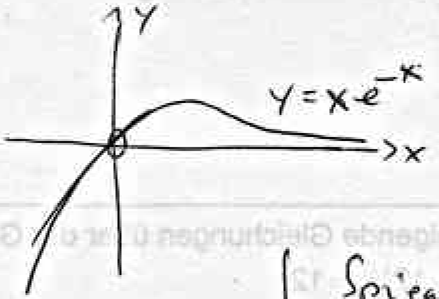
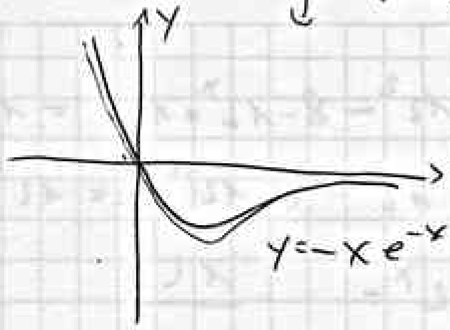
5. $f(x) = x e^{-x}$ a)



Spiegelung x-Achse

1)



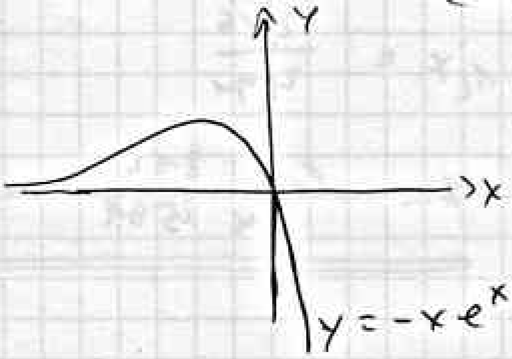
$y \to -y$

$-y = x e^{-x}$

$y = -x e^{-x}$

⟵ Spiegelung y-Achse

2)

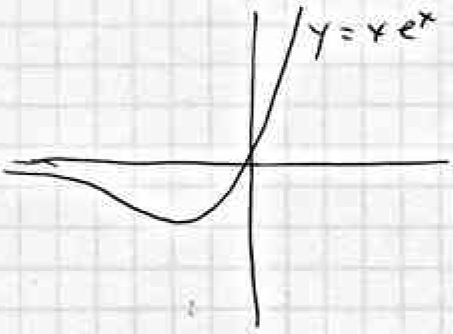


$x \to -x$

$y = -x e^{+x}$

Punktspiegelung (0|0)

3)



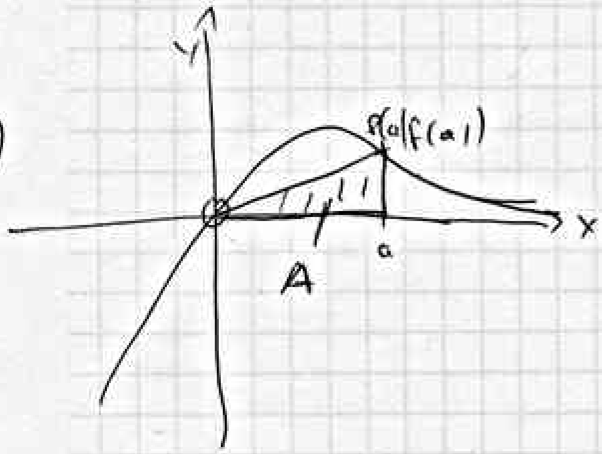
$x \to -x$

$y \to -y$

$-y = -x e^{x}$

$y = x e^{x}$

b)



$A = \frac{1}{2} a \cdot f(a) = \frac{1}{2} a \cdot a \cdot e^{-a} = \text{Zielf.}$

$A(a) = \frac{1}{2} a^2 e^{-a}$ $\quad \mathbb{D} =]0; \infty[$

$A'(a) = a e^{-a} + \frac{1}{2} a^2 e^{-a} (-1)$

$= a e^{-a} (1 - \frac{1}{2} a)$

$A'(a) = 0$

$\underline{a = 2}$ $\quad A(2) = 2 e^{-2}$

$(a = 0) \notin \mathbb{D}$

Ränder: $\lim_{a \to \infty} A(a) = 0$ $\quad$ Exp.-Fkt stärker als Potenz.

$\lim_{a \to 0} A(a) = 0$

VZT

a	1	2	3
A'(a)	+	0	–
	↗	→	↘
		Max	

also gibt a = 2 ein Dreieck maximaler Fläche