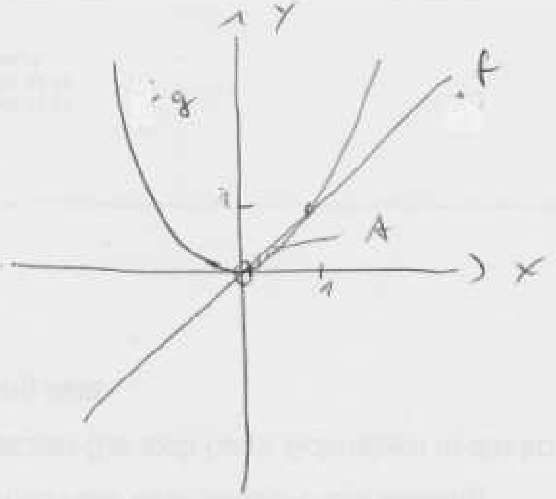
Mater H 12

1. a)



Schnittpunkte: $x = 1$ $y = 1$ $S_1(\cancel{0}1|1)$

$x = 0$ $y = 0$ $S_2(0|0)$

Schnittwinkel: S_2: 45° Steigung $\left.\begin{array}{l} f: 1, \text{ also } 45° \\ g: 0, \text{ Schnittel} \end{array}\right\} 45°$

S_1: $g'(x) = 2x$

$g'(1) = 2 = \tan \alpha_1$

$\alpha_1 = 63{,}43°$ Schnittwinkel: $\underline{18{,}43°}$

b) $A = \int_0^1 (g - f)\,dx = \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \underline{\frac{1}{6}}$

c)

$g = f$

$ax^2 = x$

$x(ax - 1) = 0$

$x_1 = 0$

$x_2 = \frac{1}{a}$

$6 = \int_0^{\frac{1}{a}} (f - g)\,dx = \left[\frac{1}{2}x^2 - \frac{a}{3}x^3\right]_0^{\frac{1}{a}}$

$6 = \frac{1}{2a^2} - \frac{1}{3a^2}$

$a = \pm\frac{1}{6}$

$\underline{a = +\frac{1}{6}}$

Mathe H12

2a) 1) Normalenvektor zu g: $\vec{n} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Normale durch O: $n: \vec{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot s$

Schnitt g und n → Lotfusspunkt F

$$g = n$$

$$\begin{aligned} 13 + t &= 2s \\ 4 - 2t &= s \end{aligned}$$

$s = 6$; $t = 1$ $\quad \underline{F(12|6)}$

2)

$$\begin{aligned} 12 &= x \cdot 5 + y \cdot 6 \\ 6 &= x \cdot (-1) + y \cdot 2 \end{aligned}$$

$x = -\frac{3}{4}$; $y = \frac{21}{8}$

$$\underline{\underline{\overrightarrow{OF} = -\frac{3}{4}\overrightarrow{OA} + \frac{21}{8}\overrightarrow{OB}}}$$

b) $\frac{A_1}{A_2} = \frac{4}{1} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 \Rightarrow \underline{\frac{r_1}{r_2} = \frac{2}{1}}$

T teilt M_1M_2 im Verhältnis 2 : 1

$$\vec{r}_T = \frac{2\vec{r}_{M_2} + 1\vec{r}_{M_1}}{2+1} = \frac{2 \cdot \begin{pmatrix} 4 \\ 12 \end{pmatrix} + \begin{pmatrix} -5 \\ 0 \end{pmatrix}}{3} = \frac{\begin{pmatrix} 3 \\ 24 \end{pmatrix}}{3} = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$$

$\underline{\underline{T(1|8)}}$

Tangente durch T ⊥ $\overrightarrow{M_1M_2} = \begin{pmatrix} 9 \\ 12 \end{pmatrix} \sim \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$$\vec{n}_{M_1M_2} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

$$\underline{\underline{t: \vec{x} = \begin{pmatrix} 1 \\ 8 \end{pmatrix} + s \cdot \begin{pmatrix} -4 \\ 3 \end{pmatrix}}}$$

3. a) $\vec{AB} = \begin{pmatrix} 8 \\ 3 \\ -5 \end{pmatrix}$ $\quad AB^2 = 98$

$\vec{AC} = \begin{pmatrix} 12 \\ -6 \\ -4 \end{pmatrix}$ $\quad AC^2 = 196$ $\quad$ = gleichschenklig

$\vec{BC} = \begin{pmatrix} 4 \\ -9 \\ 1 \end{pmatrix}$ $\quad BC^2 = 98$

$\underline{AB^2 + BC^2 = AC^2}$ Pythagoras -> rechtwinklig

b)

D muss: in E(ABC) liegen $\quad : \mathrm{I} \quad \vec{n} \cdot \vec{CD} = 0$

$\vec{CD} \perp \vec{CA}$ $\quad : \mathrm{II} \quad \vec{CD} \cdot \vec{CA} = 0$

$CD = CA$ $\quad : \mathrm{III} \quad CD = CA$

$\vec{n}_{ABC} = \vec{AB} \times \vec{BC} = \begin{pmatrix} 8 \\ 3 \\ -5 \end{pmatrix} \times \begin{pmatrix} 4 \\ -9 \\ 1 \end{pmatrix} = \begin{pmatrix} -42 \\ -28 \\ -84 \end{pmatrix} \sim \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}$

$\vec{CD} = \begin{pmatrix} x-8 \\ y+2 \\ z-1 \end{pmatrix}$

I. $3(x-8) + 2(y+2) + 6(z-1) = 0$

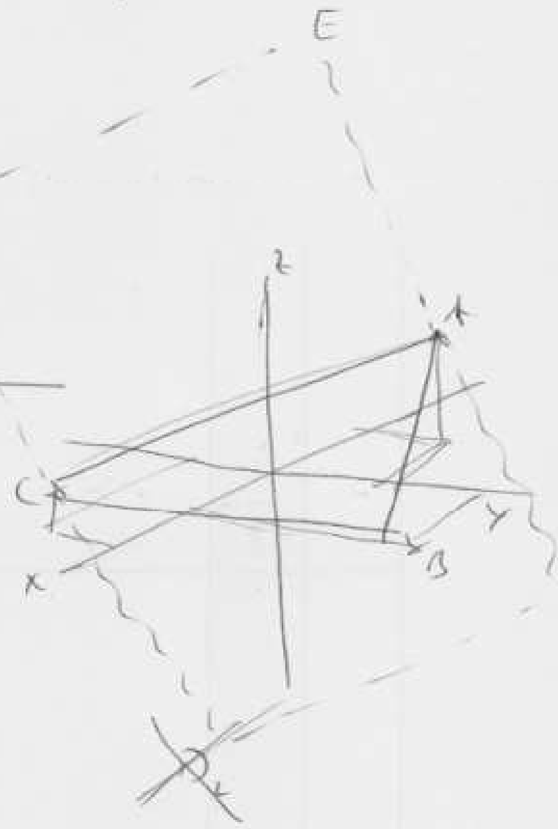
II. $12(x-8) - 6(y+2) - 4(z-1) = 0$

III. $(x-8)^2 + (y+2)^2 + (z-1)^2 = 196$

D

$\underline{x_1 = 4 \; ; \; y_1 = -14 \; ; \; z_1 = 7}$

$x_2 = 12 \; ; \; y_2 = 10 \; ; \; z_2 = -5$ $\quad$ } ?



$\underline{D(4|-14|7)}$

$\vec{AE} = \vec{CD} = \begin{pmatrix} -4 \\ -12 \\ 6 \end{pmatrix}$

$\underline{E(-8|-8|-2)}$

c) $\vec{AD} = \begin{pmatrix} 8 \\ -18 \\ 2 \end{pmatrix}$ $\quad AD^2 = 392$

$\vec{BD} = \begin{pmatrix} 0 \\ -21 \\ 7 \end{pmatrix}$ $\quad BD^2 = 490$

$AD^2 + AB^2 = BD^2 \Rightarrow \perp$ bei A

$\tan\alpha = \frac{98}{392} = \frac{1}{4}$

$\underline{\alpha = 14°}$

Mahr H12

4. a)

A	B
1	–
2	(1,1)
3	(1,2) ; (2,1)
4	(1,3) ; (3,1) ; (2,2)
5	(2,3) ; (3,2) ; (4,1) ; (1,4)
6	(1,5) ; (5,1) ; (2,4) ; (4,2) ; (3,3)
$\frac{1}{6}$	je $\frac{1}{36}$

$\left.\right\}$ 15

$P(\text{Unentschieden}) = 15 \cdot \frac{1}{6} \cdot \frac{1}{36} = \underline{\underline{\frac{15}{216}}} \approx 6{,}94\,\%$

b)

A	B
3	(1,1)
4	(1,2) ; (2,1) ; (1,1)
5	(1,1) ; (1,2) ; (2,1) ; (3,1) ; (1,3) ; (2,2)
6	(1,1) ; (1,2) ; (2,1) ; (2,2) ; (3,1) ; (1,3) ; (1,2) ; (1,4) ; (3,2) ; (2,3)

$\left.\right\}$ 20

$P(\text{A gewinnt}) = \underline{\underline{\frac{20}{216}}} = \frac{5}{54}$

c) $P(\text{A gewinnt mindestens einmal}) = 1 - P(\text{A gewinnt nie})$

$= 1 - \left(\frac{49}{54}\right)^{10}$

$\underline{\underline{= 62{,}2\,\%}}$

(Mater Ht12

5, a) $\text{I.}\quad a \cdot e^{-\frac{1}{2}\ln 4} - b = 0$

$\text{II}\quad a \cdot e^{-\frac{1}{2}0} - b = 1$

$\text{I.}\quad \frac{a}{e^{\ln 4^{\frac{1}{2}}}} - b = 0$

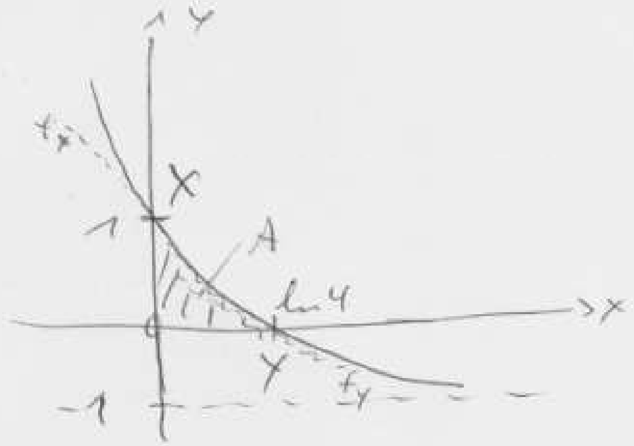
$\text{II}\quad a - b = 1$

$\text{I}\quad \frac{a}{2} - b = 0$

$\text{II}\quad a - b = 1$

$\underline{\underline{a = 2 \;;\; b = 1}}$

b) $f(x) = 2e^{-\frac{1}{2}x} - 1$



$A = \int_0^{\ln 4} (2e^{-\frac{1}{2}x} - 1)\, dx$

$= \left[-4e^{-\frac{1}{2}x} - x\right]_0^{\ln 4} = -4e^{-\frac{1}{2}\ln 4} - \ln 4 - (-4e^{-\frac{1}{2}0} - 0)$

$= -2 - \ln 4 + 4$

$= \underline{\underline{2 - \ln 4}}$

c) $f'(x) = -e^{-\frac{1}{2}x}$

$f'(0) = -1 \quad : X \qquad = m_1$

$f'(\ln 4) = -e^{-\frac{1}{2}\ln 4} = -\frac{1}{2} = m_2$

$\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right| = \frac{1}{3}$

$\underline{\underline{\alpha = 18{,}4^\circ}}$