Mat GFH14

1. $f(x) = \frac{p}{x} + \frac{q}{x^2}$; $x > 0$ $p, q \in \mathbb{R}$

a) I $f(2) = 1$

II. $f'(2) = 0$

$f'(x) = -\frac{p}{x^2} - 2\frac{q}{x^3}$

I. $\frac{p}{2} + \frac{q}{4} = 1$

II. $-\frac{p}{2^2} - 2\frac{q}{8} = 0$

$\Rightarrow p = 4$, $q = -4$

$f(x) = \frac{4}{x} - \frac{4}{x^2}$

b) $\lim_{|x| \to \infty} f(x) = 0$: x-Achse waag. A.

$f'(x) = -\frac{4}{x^2} + \frac{8}{x^3}$

$f''(x) = \frac{8}{x^3} - \frac{24}{x^4}$

$f'''(x) = -\frac{24}{x^4} + \frac{96}{x^5}$

NST. $\frac{4x-4}{x^2} = 0$

$x = 1$

WP. $\frac{8x-24}{x^4} = 0$

$x = 3$

$y = 8/9$

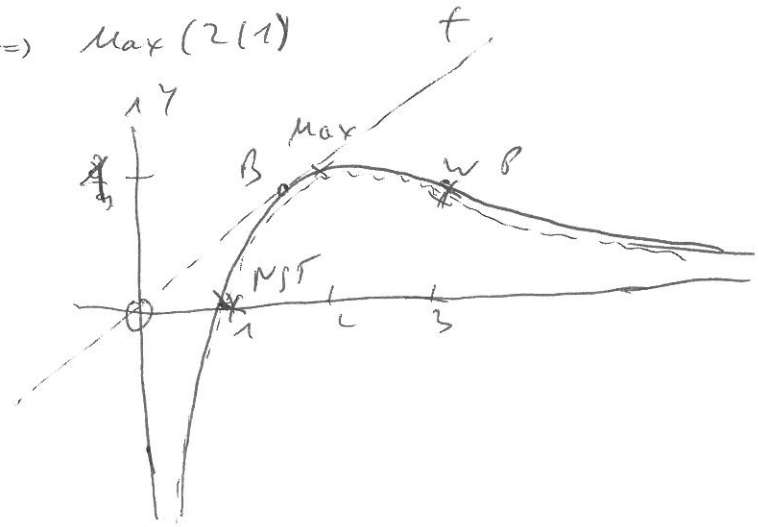
$f'''(3) \neq 0$ WP $(3 | 8/9)$

Ext. $\frac{-4(x+8)}{x^3} = 0$

$x = 2$

$y = 1$

$f''(2) < 0 \Rightarrow$ Max $(2|1)$



c) $\frac{f(x) - 0}{x - 0} = f'(x)$

$\frac{\frac{4}{x} - \frac{4}{x^2}}{x} = -\frac{4}{x^2} + \frac{8}{x^3}$

$\frac{4}{x^2} - \frac{4}{x^3} = -\frac{4}{x^2} + \frac{8}{x^3}$

$\frac{8}{x^3} = \frac{12}{x^3}$

$x = \frac{12}{8} = \frac{3}{2}$

$y = 8/9$

$B(\frac{3}{2} | \frac{8}{9})$

Mathe GFH1

2.1. $f_a(x) = \frac{1}{4}x^4 - \frac{a}{2}x^2 - \left(\frac{a}{2}\right)^2 \quad a \in \mathbb{R} \setminus \{0\}$

$f_a'(x) = x^3 - ax$

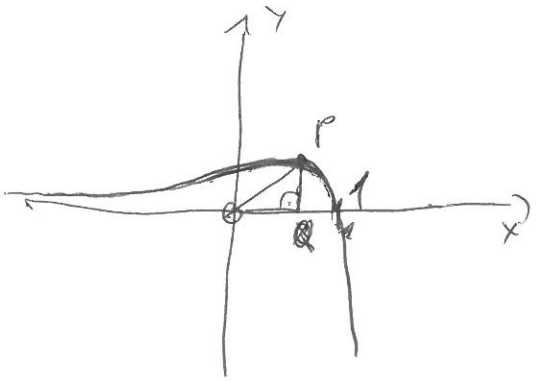
$f''(x) = 3x^2 - a = 0$

$x^2 = \pm\sqrt{\frac{a}{3}}$ nur möglich für $a > 0$

$f'''(x) = 6x \qquad f''' \neq 0$ für $x = \pm\sqrt{\frac{a}{3}}$, also WP für $a > 0$

kein WP für $a < 0$

2.2. $g(x) = (1-x)e^x$



$A = \frac{1}{2} x \cdot f(x)$

$A(x) = \frac{1}{2}x(1-x)e^x$

$= \frac{1}{2}(x - x^2)e^x$

$A'(x) = \frac{1}{2}(1-2x)e^x + \frac{1}{2}(x-x^2)e^x$

$= \frac{1}{2}(1 - 2x + x - x^2)e^x$

$A'(x) = \frac{1}{2}\underbrace{(1 - x - x^2)}_{=0}\underbrace{e^x}_{\neq 0} = 0$

$x = \frac{-1 \pm \sqrt{5}}{2} \qquad x = \frac{-1+\sqrt{5}}{2}$

Ränder: $A(0) = 0$

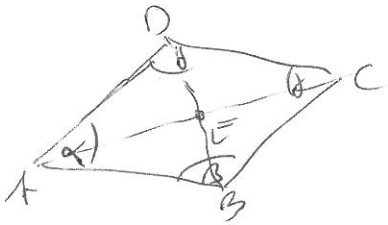
$A(1) = 0$

VZT	0,5	x_1	1
$f'(x)$	+	0	−
	↗	Max	↘

$x = \frac{-1+\sqrt{5}}{2}$

Mathe GF WH 14

3. $A(-4|5|-2)$ $B(6|3|9)$ $E(-6|5|6)$



a)

$$\vec{r}_C = \vec{r}_A + 2\vec{AE} = \vec{r}_A + 2(\vec{r}_E - \vec{r}_A) = 2\vec{r}_E - \vec{r}_A = \begin{pmatrix} -8 \\ 5 \\ 14 \end{pmatrix}$$

$C(-8|5|14)$

$$\vec{r}_D = \vec{r}_B + 2\vec{BE} = \begin{pmatrix} -18 \\ 7 \\ 3 \end{pmatrix}$$

$D(-18|7|3)$

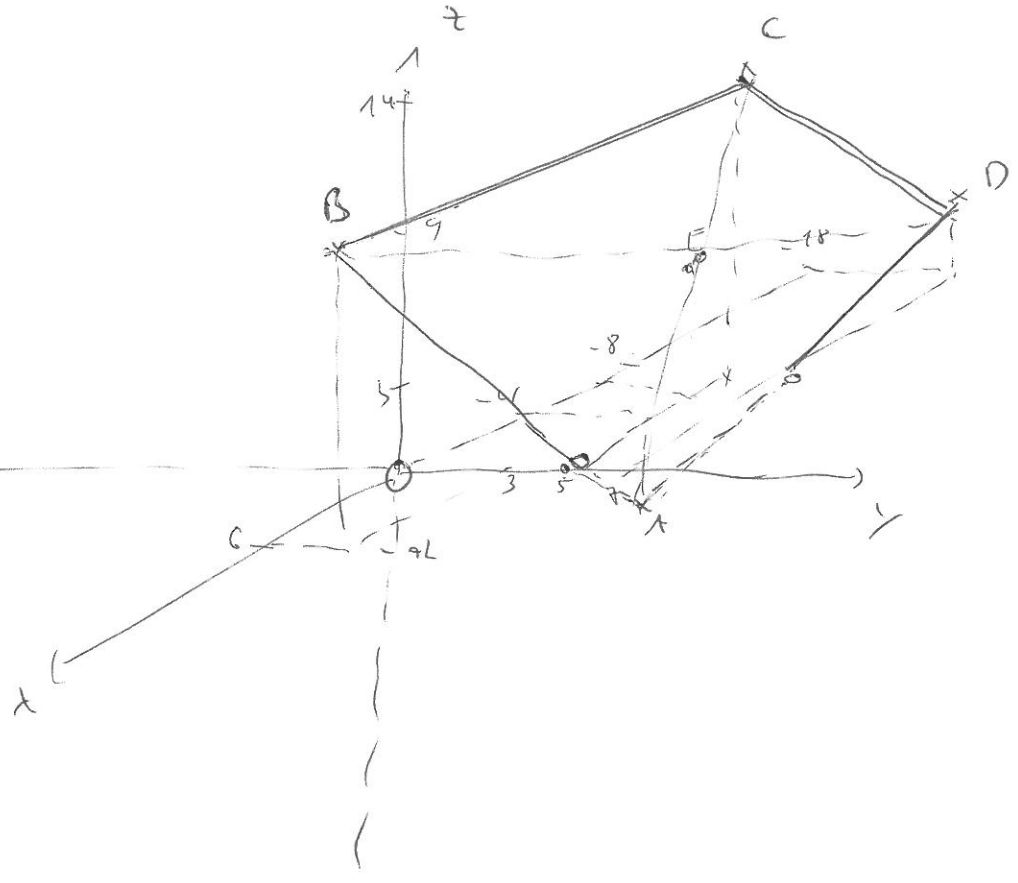
b) $\alpha = \gamma$; $\beta = \delta$; $\alpha + \beta = 180°$

$$\cos\alpha = \frac{\vec{AB} \cdot \vec{AD}}{|AB| \cdot |AD|} = \frac{\begin{pmatrix} 10 \\ -2 \\ 11 \end{pmatrix}\begin{pmatrix} -14 \\ 2 \\ 5 \end{pmatrix}}{15 \quad 15} = \frac{-89}{255} \Rightarrow \alpha = 113{,}3° = \gamma$$

$\beta = \delta = 66{,}7°$

$A = a \cdot b \cdot \sin\alpha = 15^2 \cdot \sin 113{,}3° = 206{,}7$

c) $g: \vec{X} = \begin{pmatrix} -4 \\ 5 \\ -2 \end{pmatrix} + t \cdot \begin{pmatrix} 10 \\ -2 \\ 11 \end{pmatrix}$



Mat GF 14

4. 4g | 8b | 12r : 24

a) $P = \left(\frac{4}{24}\right)^2 + \left(\frac{8}{24}\right)^2 + \left(\frac{12}{24}\right)^2 = \frac{7}{18} = 38{,}9\,\%$

b) (gbr): 6

$P = \frac{4 \cdot 8 \cdot 12}{24 \cdot 23 \cdot 22} \cdot 6 = \frac{48}{253} = 19\,\%$

c) +12 : 2r , x g , (20 − x) b | 14r | (4 + x) g | (8 + 20 − x) b |

$P(gb) = \frac{4}{27} = 2 \cdot \frac{(4+x)(28-x)}{36^2}$

$x = 2$
$(x = 12)$

2 gelbe, 8 blaue

MatGF W14

5.1. $x = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow$

$M(3|2)$

$y = 1 \cdot (x-4) + 0$

$y = x - 4$

HNF: $\frac{x - y - 4}{\sqrt{2}} = 0$

$M(3|2)$: $\left| \frac{3 - 2 - 4}{\sqrt{2}} \right| = \frac{3}{\sqrt{2}} = \frac{3}{2}\sqrt{2} = R$

5.2. $k: (x-3)^2 + (y-2)^2 = \frac{9}{2}$

x-Achse: $(x-3)^2 + 4 = \frac{9}{2}$

$\underline{X = 3 \pm \sqrt{\frac{1}{2}} = 3 \pm \frac{1}{2}\sqrt{2}}$

y-Achse: $9 + (y-2)^2 = \frac{9}{2}$

$(y-2)^2 = -\frac{9}{2} < 0$ kein Schnitt

5.2. a) $N(t) = N_0 \cdot 1{,}035^t$

$N(2) = N_0 + 100\,000 = N_0 \cdot 1{,}035^2$

$\underline{N_0 = \frac{100\,000}{1{,}035^2 - 1}} = \underline{1'404'001}$

b) $2{,}2\Gamma = 1{,}035^x$

$\underline{x = \log_{1{,}035} 2{,}2\Gamma} = \underline{23{,}57 \text{ (Jahre)}}$