MH16

1. $f(x) = c + 2x^2 - 4x^4$; $c \in \mathbb{R}$ $\quad f(-x) = f(x)$: Achsensymmetrie

a) $c = 0$ $\quad f(x) = 2x^2 - 4x^4$ $\quad$ NST: $x = 0$ doppelt $\quad x = \pm\frac{1}{2}\sqrt{2}$ einfach

$f'(x) = 4x - 16x^3$ $\quad$ Ext: $x = 0$; $x = \pm\frac{1}{2}$; $f''(0) > 0 \Rightarrow Min(0|0)$

$f''(\frac{1}{2}) < 0 \Rightarrow Max(\pm\frac{1}{2}|\frac{1}{4})$

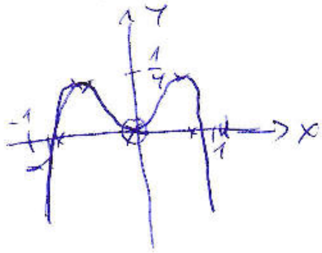
$f''(x) = 4 - 48x^2$ $\quad$ WP: $x = \pm\frac{1}{6}\sqrt{3}$ einfach

$WP(\pm\frac{1}{6}\sqrt{3}|\frac{5}{36})$

b) $y_{HP} = \frac{1}{4} \Rightarrow c = -\frac{1}{4}$

$A = -\int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)dx = -2\int_0^{\frac{1}{2}} f(x)dx = -2[-\frac{1}{4}x + \frac{2}{3}x^3 - \frac{4}{5}x^5]_0^{\frac{1}{2}} = \frac{2}{15}$

Symm.



c) $f(1) = 0$

$c + 2 - 4 = 0$

$c = 2$

2. a) $E: \vec{x} = \begin{pmatrix}-4\\5\\9\end{pmatrix} + s\begin{pmatrix}20\\5\\-15\end{pmatrix} + t\begin{pmatrix}14\\5\\-2\end{pmatrix}$ $\quad$ ($\vec{AB}$, $\vec{AC}$) $\quad g: \vec{x} = \begin{pmatrix}2\\-8\\9\end{pmatrix} + r\begin{pmatrix}-12\\21\\-27\end{pmatrix}$

b) $\vec{AB} \times \vec{AC} = \begin{pmatrix}85\\-170\\170\end{pmatrix} \sim \begin{pmatrix}1\\-2\\2\end{pmatrix} = \vec{n}$ $\quad \vec{n}\cdot\vec{PQ} = -96 \Rightarrow E \nparallel g$ also Schnitt

$E: x - 2y + 2z + d = 0 \xrightarrow{A} d = -4 \Rightarrow E: x - 2y + 2z - 4 = 0$

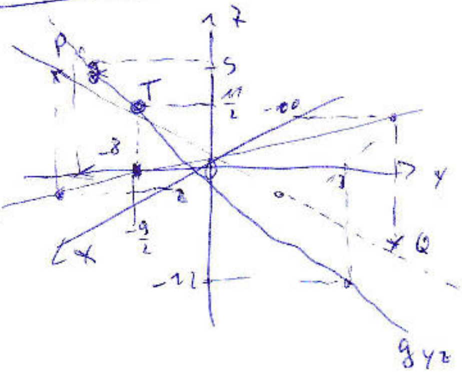
$g \cap E: (2 - 12r) + 2(-8 + 21r) + 2(9 - 27r) - 4 = 0 \Rightarrow r = \frac{1}{3}$ $\quad S(-2|-1|2)$

c) $AB^2 = 650$ $\quad AC^2 = 225$ $\quad BC^2 = 305$

$\cos\gamma = \frac{\vec{AC}\cdot\vec{BC}}{AC \cdot BC} = \frac{-60}{15\sqrt{305}} = -0{,}229 \Rightarrow \gamma = 103{,}2°$

d) g_{xy}: $(-2|-8)$; $(-10|13)$ $\quad y = -\frac{7}{4}x - \frac{9}{2}$

g_{yz}: $(13|-12)$; $(-8|9)$ $\quad z = 1 - y \rightarrow z = \frac{11}{2}$ $\quad T(0|-\frac{9}{2}|\frac{11}{2})$



3 a) $F(x) = (a - x)e^x$ $\quad F'(x) = -1e^x + (a - x)e^x = (a - 1 - x)e^x$

$f(x) = (1 - x)e^x$

$a = 2$

$A = \int_0^1 (1 - x)e^x dx = [(2 - x)e^x]_0^1 = e - 2$

b) $\sin u = 2\cos u$

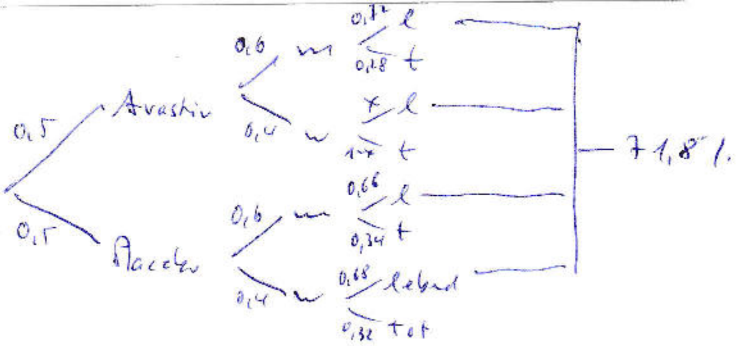
$\tan u = 2$

$u = 1{,}11 + z\cdot\pi$

$3x + \frac{\pi}{4} = 1{,}11 + z\cdot\pi$

$x = 0{,}107 + z\cdot\frac{\pi}{3}$ $\quad \mathbb{L} = \{-0{,}94 ; 0{,}11 ; 1{,}15\}$

4. a)



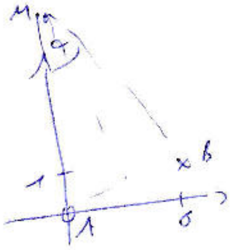
b) $P(P \wedge m \wedge l) = 0{,}5 \cdot 0{,}6 \cdot 0{,}66 = 19{,}8\,\%$

c) $P(m \wedge t) = 0{,}5 \cdot 0{,}28 + 0{,}5 \cdot 0{,}34 = 31\,\%$

c) $0{,}718 = 0{,}5 \cdot 0{,}6 \cdot 0{,}72 + 0{,}5 \cdot 0{,}4 \cdot x + 0{,}5 \cdot 0{,}6 \cdot 0{,}66 + 0{,}5 \cdot 0{,}4 \cdot 0{,}68$

$x = 84\,\%$

5. a)



$MA = MB$

$\left|\begin{pmatrix} 0-0 \\ 0-y \end{pmatrix}\right| = \left|\begin{pmatrix} 8-0 \\ 1-y \end{pmatrix}\right|$

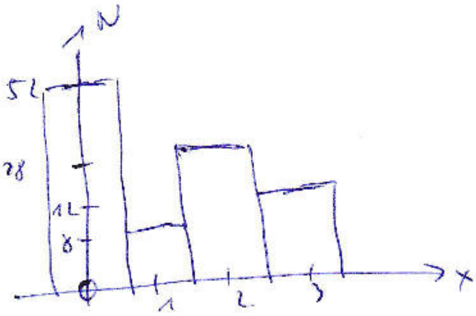
$y^2 = 64 + (1-y)^2$

$y = 65/2$

$M(0|\frac{65}{2})$ $\quad R = \frac{65}{2}$

$\cos\alpha = \frac{\vec{MA} \cdot \vec{MB}}{|MA|\,|MB|} = \frac{4095/4}{4225/4} \Rightarrow \alpha = 14{,}25°$

b)



$\overline{x} = \frac{1}{100} \cdot (52 \cdot 0 + 1 \cdot 8 + 2 \cdot 28 + 3 \cdot 12) = 1$

$\sigma^2 = \overline{x^2} - \overline{x}^2 = 1{,}28$

$\sigma = \sqrt{\sigma^2} = 1{,}13$

Median: bei einer geraden Anzahl von Werten ist es der Mittelwert der beiden "mittleren" der sortierten Abfolge.
Da hier die ersten 52 Werte gleich 0 sind, sind die beiden mittleren, also der 50ste und 51ste Wert auch Null, somit auch ihr Mittelwert.
Median = 0