Matur GTR #18

1. a) f: P.S., dann ungerade Exp.

NST: $x=0$ einfach, $x=\pm\sqrt{3}k$ einfach

$f_k'(x) = k^2 - x^2$: Extr. $x=\pm k$

g: A.S., nun gr. Exp.

NST, $x=\pm 2$ einfach

$g_3(x) = -2kx^4$: Extr. $x=0$ abs. max, da Parabel $\cap$

b) $f(3) = g(3)$

$3k^2 - 9 = k(4-9)$

$k_{1,2} = \frac{-5 \pm \sqrt{133}}{6}$; $k_2 < 0$ fällt weg

c) $f_3(x) = 9x - \frac{1}{3}x^3$

$f_3'(x) = 9 - x^2$

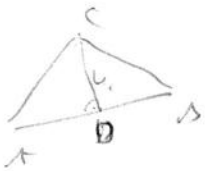
$f_3''(x) = -2x = 0 \rightarrow f_3'(0) = 9$, $f(0) = 0$ $\rightarrow$ $t: y = 9x$

$t = g_3$

$9x = 3(4-x^4)$

$x = -4$ $x = 1$

2. a) $\cos\beta = \frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{BA \cdot BC} = \frac{\binom{12}{5}\binom{4}{1}}{13 \cdot 5\sqrt{17}} = \frac{75}{75\sqrt{17}} = \frac{1}{\sqrt{17}} \Rightarrow \beta = 63{,}4°$



b) $h_c = BC \cdot \sin\beta = 5\sqrt{17} \cdot \cos 63{,}4 = 10$ ($\cos\beta = \frac{2}{\sqrt{17}}$)

$\cos^2\beta + \sin^2\beta = 1$

$\sin\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-\frac{4}{17}} = \frac{1}{\sqrt{17}}$

c) $\overrightarrow{AD} \perp \overrightarrow{AB} = \begin{pmatrix}-12\\-5\end{pmatrix} \xrightarrow{D} \begin{pmatrix}5\\-12\end{pmatrix} \sim DC$

$\perp \begin{pmatrix}a\\+12\end{pmatrix} \sim \overrightarrow{DC}$ ($11 = 15$)

$\frac{1}{2}\overrightarrow{CD} = \frac{10}{30}\begin{pmatrix}-9\\+12\end{pmatrix} = \begin{pmatrix}-3\\+4\end{pmatrix}$ $\quad \vec{r}_C + \frac{1}{2}\overrightarrow{CD} = \begin{pmatrix}7\\4\end{pmatrix} + \begin{pmatrix}-3\\4\end{pmatrix} = \vec{0}$

d) $k: x^2 + y^2 = 5^2$ $\quad AC: \vec{x} = \begin{pmatrix}5\\10\end{pmatrix} + t\begin{pmatrix}1\\7\end{pmatrix}$

x-Achse: $y = \pm 5$

y-Achse: $x = \pm 5$

$AC \cap k$: $(5+t)^2 + (10+7t)^2 = 5^2$

$t_1 = -2 \rightarrow S_1(3|-4) = C$

$t_2 = -1 \rightarrow S_2(4|3)$

3. a) $P(K_0 \cap f) = \frac{4}{5}\cdot\frac{1}{2} + \frac{1}{5}\cdot 1 = \frac{3}{5} = 60\%$

b) $\frac{1/5}{3/5} = \frac{1}{3} = 33{,}3\%$

c) $P(3\times \text{Kopf}) = \frac{4}{5}\left(\frac{1}{2}\right)^3 + \frac{1}{5}\cdot 1^3 = \frac{3}{10} = 30\%$

d) i) $P(\text{gef. dabei}) = 2\cdot\frac{4}{5}\cdot\frac{1}{4} = \frac{2}{5} = 40\%$

ii) $P(\text{"KE mit 2 no-ak"}) + P(\text{"KE mit mind 1 gef"}) = 0{,}6\cdot\frac{1}{4}\cdot 2 + 0{,}4\cdot\frac{1}{2} = 50\%$

iii) $P(\text{"kl" ...}) > 99{,}95\%$

$0{,}6\left(1-\left(\frac{1}{2}\right)^n\right) + 0{,}4\left(1-\left(\frac{1}{2}\right)^n\right) > 0{,}9995$

$\left(\frac{1}{2}\right)^n < 0{,}0005$

$n > \ln 0{,}0005 : \ln\frac{1}{2} = 13{,}28$ ab 14 Würfe

Matu 6F
H18

4.1. a) $q = \sqrt[5]{\frac{128,8}{111}} = 1,028355 \rightarrow \underline{2,835\,\%}$

b) $128,8 : 1,03 = 125,0486$ $\quad q = \sqrt[4]{\frac{125,04}{111}} = 1,0279 \rightarrow \underline{2,79\,\%}$

4.2. $f'\left(\frac{\pi}{4}\right) = 1$; $\quad f'(x) = 2\cos x\,(-\sin x) + a\cos x$

$= -2\sin x\cos x + a\cos x$

$-2 \cdot \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} + a\frac{\sqrt{2}}{2} = 1$

$\underline{a = 2\sqrt{2}}$

4.3 $\bar{x} = \underline{6600}$ $\quad s = \underline{\frac{1717}{\text{[illegible]}}}$

$\tilde{x} = (5300 + 6750)/2 = \underline{6025}$

4.4. a) $\vec{u} \cdot \vec{v} = 0$

$-x^2 - x + 6 = 0$

$\underline{x = -3}$

$\underline{x = 2}$

b) $-x^2 - x + 6 \rightarrow \max$ Scheitel $\underline{x = -\frac{-1}{2(-1)} = -\frac{1}{2}}$

4.5 $F = \int_0^{100} \left(x - 0,1 \cdot x^{1,5}\right) dx = \left[\frac{1}{2}x^2 - \frac{1}{2,5}x^{3,5}\right]_0^{100} = 1000$ $\quad \left\{ \underline{\frac{F}{A} = 20\,\%} \right.$

$A_{ABC} = 5000$