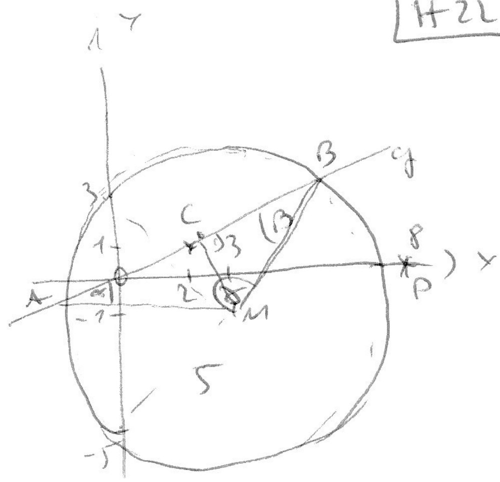
GF
H22

1. $x^2-6x+y^2+2y=25$

a) $x^2-6x+9+y^2+2y+1=25+9+1$

$(x-3)^2+(y+1)^2=25$

$\underline{M(3|-1)}$ $\underline{R=5}$



b) $P(8|0)$ in k: $(8-3)^2+(0+1)^2=25$

$26=25$ (f)

$\Rightarrow$ $\underline{\text{ausserhalb.}}$

y-Achse: $x=0$: $(0-3)^2+(y+1)^2=25$

$\underline{y_1=3}$

$\underline{y_2=-5}$

c) g: $y=\frac{1}{2}x$ in k: $(2y-3)^2+(y+1)^2=25$

$x=2y$

$y=-1 \to x=-2$

$y=3 \to x=6$

$\underline{A(-2|-1)}$

$\underline{B(6|3)}$ $\Big\}$ $AB=\sqrt{80}$

$BC=2\sqrt{5}$

$\cos\beta=\frac{2\sqrt{5}}{5} \to \underline{\beta=26{,}57°}$

$\underline{\alpha=\beta}$

$\gamma=180°-2\beta$

$\underline{=126{,}9°}$

$\vec{MA}=\begin{pmatrix}-5\\0\end{pmatrix}$ $\begin{pmatrix}-5\\0\\0\end{pmatrix}\times\begin{pmatrix}3\\4\\0\end{pmatrix}=\begin{pmatrix}0\\0\\-20\end{pmatrix}$

$\vec{MB}=\begin{pmatrix}3\\4\end{pmatrix}$

$\underline{A_{AMB}}=\frac{1}{2}|\vec{MA}\times\vec{MB}|=\underline{10}$

2. $f(x)=\frac{1}{2}e^{-2x^2}$

a) Exponentialfunktion haben keine Nullstellen.

$f(-x)=\frac{1}{2}e^{-2(-x)^2}=\frac{1}{2}e^{-2x^2}=f(x)$: Achsensymmetrie zur y-Achse

b) $\lim_{x\to\pm\infty}\frac{1}{2}e^{-2x^2}=0$: x-Achse ist waag. A.

$f'(x)=\frac{1}{2}e^{-2x^2}(-4x)=-2x\,e^{-2x^2}$

$f'=0$

$\underline{x=0}$

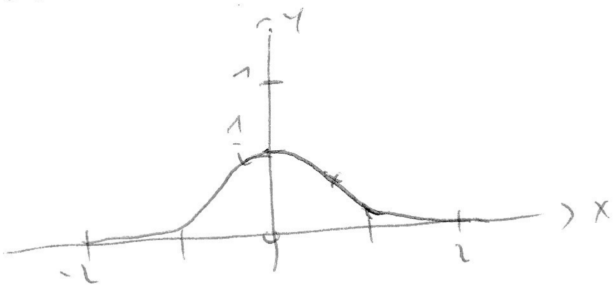
$y=\frac{1}{2}$ $f''(0)<0 \Rightarrow \underline{Max(0|\frac{1}{2})}$

$f''(x)=-2(e^{-2x^2}+x\,e^{-2x^2}(-4x))=-2(1-4x^2)e^{-2x^2}$

$f''=0$

$\underline{x=\pm\frac{1}{2}}$ einfach, VZW, WP

$\underline{y=\frac{1}{2}e^{-\frac{1}{2}}}$ $WP(\pm\frac{1}{2}|\frac{1}{2\sqrt{e}})$



c) $f(x)=\frac{1}{k}\cdot e^{-kx^2}$

$f'(x)=-2x\,e^{-kx^2}$

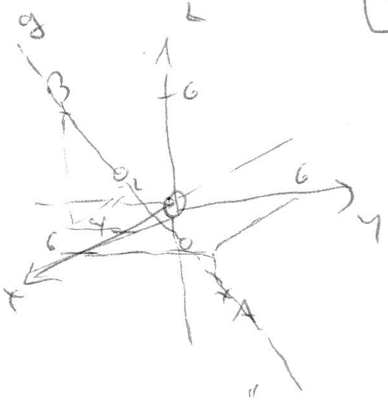
$f''(x)=-2(e^{-kx^2}+x\,e^{-kx^2}(-2kx))=-2\underbrace{(1-2kx^2)}_{=0}\underbrace{e^{-kx^2}}_{\neq 0}=0$

$x=\pm\sqrt{\frac{1}{2k}}$ $k>0$

Für $k\leq 0$ kein WP.

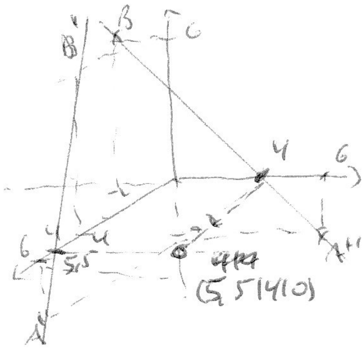
GF
1+21

3. g: $\vec{x} = \underset{A}{\begin{pmatrix}6\\6\\-2\end{pmatrix}} + t\underset{\vec v}{\begin{pmatrix}1\\4\\-4\end{pmatrix}}$



a) $\vec r_B = \begin{pmatrix}6\\6\\-2\end{pmatrix} + (-2)\begin{pmatrix}1\\4\\-4\end{pmatrix} = \begin{pmatrix}4\\-2\\6\end{pmatrix}$ $\underline{B(4|-2|6)}$

b)



c) $x = 0 = 6 + t \rightarrow t = -6$: $\underline{D_{yz}(0|-18|22)}$

d) $P(88|4|112)$ $\vec u = \begin{pmatrix}2\\u\\1\end{pmatrix} \perp \vec v$: $\vec u \cdot \vec v = 0$

$\begin{pmatrix}2\\u\\1\end{pmatrix}\begin{pmatrix}1\\4\\-4\end{pmatrix} = 0$

$4u - 2 = 0$

$\underline{u = \frac{1}{2}}$

h: $\vec x = \begin{pmatrix}88\\4\\112\end{pmatrix} + m\begin{pmatrix}2\\\frac{1}{2}\\1\end{pmatrix}$

g = h
keine Lösg ; windschief

4, 4x würfeln

a) $P(4x\ '5') = (\frac{1}{6})^4 = \underline{0{,}077\%}$

b) $P(4x\ \text{'ungerade'}) = (\frac{1}{2})^4 = \underline{\frac{1}{16}}$

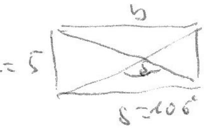
c) $P(4\ \text{verschiedene Z.}) = \frac{6\cdot5\cdot4\cdot3}{6^4} = \frac{5}{18} = \underline{27{,}8\%}$

d) $P(4\ \text{"}\ ,\ g|u|g|u) = 2\cdot\frac{3\cdot3\cdot2\cdot2}{6^4} = \frac{1}{18} = \underline{5{,}56\%}$

e) $P(\text{Summe } 22) = \underline{\frac{10}{6^4} = 0{,}77\%}$
6+6+6+4 (4)
6+6+5+5 (6)

f) $P(\underbrace{\text{gerades Prod.}}_{\text{mind. eine gerade Zahl}}) = 1 - P(\text{keine gerade Zahl}) = 1 - (\frac{1}{2})^4 = \underline{\frac{15}{16} = 93{,}75\%}$

5. a) $f(x) = \frac{x^3}{3(x-1)^2}$ $f'(x) = \frac{1}{3}\cdot\frac{3x^2(x-1)^2 - x^3\,2(x-1)}{(x-1)^4} = \frac{1}{3}\,\frac{3x^2(x-1) - 2x^3}{(x-1)^3} \underset{x=3}{=} \frac{1}{3}\,\frac{3\cdot9\cdot2 - 2\cdot27}{\ldots} = 0$ ✓

b) 

$\tan\frac{\delta}{2} = \frac{b/2}{a/2} \Rightarrow \underline{b} = a\cdot\tan\frac{\delta}{2} = 5\cdot\tan 53 = \underline{6{,}63}$

c) $f(x) = x + 3$
$g(x) = \frac{1}{4}x^3 - \frac{1}{2}x^2 + 3$

$f = g$

$\frac{1}{4}x^3 - \frac{1}{2}x^2 - x = 0$

$x(x^2 - 2x - 4) = 0$

$\underline{x=0}$ $x = 1 \pm \sqrt5$

$A = \int_{1-\sqrt5}^{0}(f-g)\,dx + \int_{0}^{1+\sqrt5}(g-f)\,dx = \left[\frac{1}{16}x^4 - \frac{1}{6}x^3 - \frac{1}{2}x^2\right]_{1-\sqrt5}^{0} + \left[\ \ldots\ \right]_0^{1+\sqrt5}$

$\underline{= \frac{13}{3} = 4{,}\overline{3}}$