NORMALES NIVEAU — MATHEMATIK — H23

1) Es gibt insgesamt $6^3 = 216$ mögliche Sequenzen.

a) $(1|1|1)$ ..... $(6|6|6)$ sind 6 Kombinationen

$$P_a = \frac{6}{216} = \underline{\underline{\frac{1}{36}}}$$

b)
654 653 652 651 643 642 641 632 631 621 | 543 542 541 532 531 521 | 432 431 421 | 321

$$P_b = \frac{20}{36} = \underline{\underline{\frac{5}{9}}}$$

c) $|z_1 + z_2| > 3 > |z_1 - z_2|$

(1 3), (3 1) | (2 2), (2 3) (3 2) (2 4) (4 2)
(3,3) (3 4), (4 3), (3,5) (5,3) | (4,5) (5,4) (4,6) (6,4) (4,4) | (5,6) (6,5) (5,5) (6,6)

$$P_c = \frac{21}{36} = \underline{\underline{\frac{7}{12}}}$$

d) Mit $z_1 = 6$ $\quad 8 \leq z_1 + z_2 + z_3 \leq 18$

$$M = \{(z_2, z_3) \mid 8 \leq z_1 + z_2 + z_3 \leq 18 \wedge \Sigma z_i \in P\}$$

$$= \{11, 13, 17\} \to z_2 + z_3 \in \{5, 7, 11\}$$

$$\to \{(5,6)(6,5), (5,2), (4,3), (3,4)(2,5), (1,4)(4,1)(2,3)(3,2)\}$$

$$P_d = \frac{10}{36} = \underline{\underline{\frac{5}{18}}}$$

e) $G = \{2,4,6\}$ $\quad U = \{1,3,5\}$

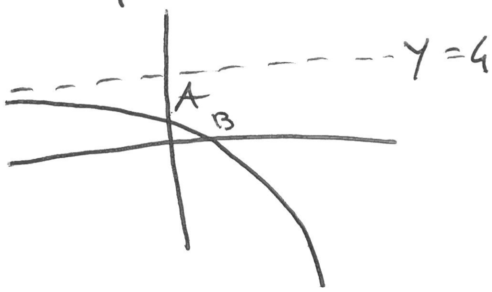
$$E = \{ z_1 \in G \vee (z_1 \in U \wedge (z_2 + z_3) \in U)\}$$

$$P_e = \frac{1}{2} + \frac{1}{2} \cdot \left(\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2}\right) = \underline{\underline{\frac{3}{4}}}$$

3) a) NS: $f(x) = 4 - 2^x = 0 \rightarrow 2^2 = 2^x \rightarrow \underline{\underline{x = 2}}$

EXTR. $f'(x) = -\ln 2 \cdot 2^x < 0$ Monoton fallend, $\underline{\underline{\text{keine Extrema}}}$

AS. $\lim\limits_{x \to +\infty} f(x) = -\infty$ $\lim\limits_{x \to -\infty} 4 - 2^x = 4$



Die Kurve kann auch mit Hilfe von affinen Transformationen skizziert werden

$2^x \xrightarrow{\text{Spiegelung}} -2^x \xrightarrow{\text{Verschiebung}} -2^x + 4$

b) ANSATZ $y = m(x - x_P) + y_P$

$x_P = 1$ $y_P = f(1) = 2$ $m = f'(1) = -2\ln 2 \simeq 1{,}39$

$y = \underline{\underline{-2\ln 2(x-1) + 2}} \simeq -1{,}39x + 3{,}39$

c) $f(0) = 4 - 2^0 = 3$ $f(2) = 4 - 2^2 = 0$ (siehe oben!)

$\underline{\underline{A, B \in \text{GRAPH}(f)}}$

$\int_0^2 f(x)\,dx = 4x - \frac{1}{\ln 2} 2^x \Big|_0^2 = \left(8 - \frac{1}{\ln 2} 2^2\right) - \left(0 - \frac{1}{\ln 2} \cdot 2^0\right)$

$= 8 - \frac{3}{\ln 2} \approx \underline{\underline{3{,}67}}$

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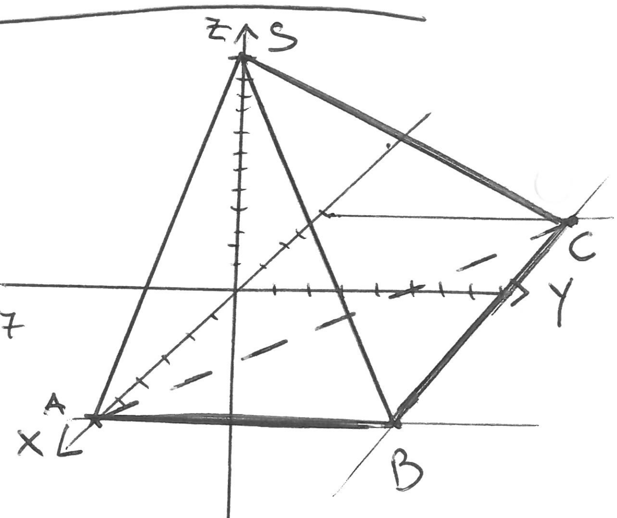
4) a) $\vec{AS} = \begin{pmatrix} -6 \\ 0 \\ 12 \end{pmatrix}$ $\vec{AB} = \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix}$

$\vec{n} = \vec{AB} \times \vec{AS} = \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix} \times \begin{pmatrix} -6 \\ 0 \\ 12 \end{pmatrix} = \begin{pmatrix} 96 \\ 0 \\ 48 \end{pmatrix}$

$A_{ABS} = \frac{1}{2} |\vec{n}| = \frac{1}{2} \sqrt{96^2 + 48^2} = 24\sqrt{5}$

$\simeq 53{,}67$



b) $E_{ABS}: \vec{r} = \vec{OA} + s\,\vec{AS} + t\,\vec{AB}$

$\vec{r} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -6 \\ 0 \\ 12 \end{pmatrix} + t \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix}$

s,t ELIMINIEREN

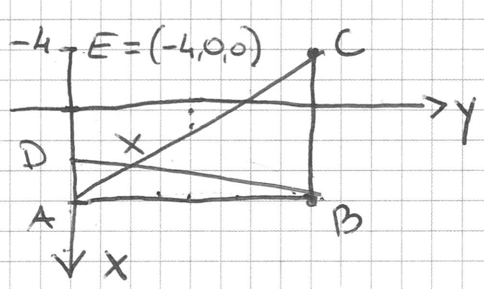
$\vec{n} \cdot (\vec{r} - \vec{r_A}) = 0 \rightarrow \begin{pmatrix} 96 \\ 0 \\ 48 \end{pmatrix} \cdot \begin{pmatrix} x - 6 \\ y - 0 \\ z - 0 \end{pmatrix} = 0$ $\boxed{2x + z - 12 = 0}$

$P \overset{?}{\in} E_{ABS}$ $2 \cdot 54 + (-72) - 12) \neq 0 \rightarrow \underline{\underline{P \notin E_{ABS}}}$

c) $\vec{r}_M = \frac{1}{3}\left(\vec{r}_A + \vec{r}_B + \vec{r}_S\right) = \frac{1}{3}\begin{pmatrix} 6+6+0 \\ 0+8+0 \\ 0+0+12 \end{pmatrix} = \begin{pmatrix} 4 \\ 8/3 \\ 4 \end{pmatrix}$

$d = |\overrightarrow{MC}| = \left|\begin{pmatrix} -4-4 \\ 8-8/3 \\ 0-4 \end{pmatrix}\right| = \sqrt{8^2 + \left(\frac{16}{3}\right)^2 + 4^2} \approx 10{,}41$

d)



$A_{ABX} : A_{XBC} = 1:3$

$\Updownarrow$

$\overline{AX} : \overline{XC} = 1:3$

$\overline{AD} : \overline{BC} = 1:3$

$\to \overline{AD} = \frac{1}{3}\,\overline{BC} = \frac{10}{3} \longrightarrow D = \left(\frac{8}{3} \middle| 0 \middle| 0\right)$

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4) $M = m_{AB} \cap g$ mit m_{AB} = Mittelsenkrechte zu AB

a) $M_{AB} = \left(\frac{23}{2}, \frac{35}{2}\right) \to m_{AB}: \vec{r} = \begin{pmatrix} \frac{23}{2} \\ \frac{35}{2} \end{pmatrix} + s\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$\vec{n}_{AB} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \perp \overrightarrow{AB} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$

$m_{AB} \cap g \to \begin{cases} x = \frac{23}{2} - s \\ y = \frac{35}{2} + 3s \\ 2x + 3y = 79 \end{cases} \longrightarrow 2\left(\frac{23}{2} - s\right) + 3\left(\frac{35}{2} + 3s\right) = 7$

$\downarrow$

$7s = \frac{7}{2}$

$\downarrow$

$s = \frac{1}{2} \longrightarrow \underline{\underline{M = (11|19)}}$

$r = |\overrightarrow{MA}| = \sqrt{(11-7)^2 + (19-16)^2} = \underline{5}$

$2x + 3y = 79 \to y = -\frac{2}{3}x + \frac{79}{3} \longrightarrow \underline{\underline{Q = \left(0 \middle| \frac{79}{3}\right)}}$

b) $K: (x-11)^2 + (y-19)^2 = 5^2$

Im A Polarisiert

$t: (x-11)(7-11) + (y-19)(16-19) = 5^2$

$\to 4x + 3y = 76$

$\to y = -\frac{4}{3}x + \frac{76}{3} \to P = \left(0 \middle| \frac{76}{3}\right)$

$\to \overline{PQ} = \left|\frac{79}{3} - \frac{76}{3}\right| = \underline{1}$

5) 1) $\sin x \cdot \cos\left(x + \frac{\pi}{3}\right) = 0$

$\sin x = 0 \rightarrow x = 0, \pi, 2\pi, \ldots$

$\cos\left(x+\frac{\pi}{3}\right) = 0 \rightarrow x + \frac{\pi}{3} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \rightarrow x = \frac{\pi}{6}, \frac{7}{6}\pi, \ldots$

$\mathbb{L} = \left\{0, \frac{\pi}{6}, \pi, \frac{7\pi}{6}, 2\pi\right\}$

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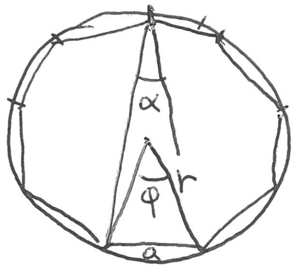
2) $\varphi = \frac{360°}{9} = 40°$



$\sin\frac{\varphi}{2} = \frac{\frac{a}{2}}{r}$

$r = \frac{a}{2\sin\frac{\varphi}{2}} = \frac{6}{2\sin 20°} \simeq 8{,}77$

$\alpha = \frac{\varphi}{2} = 20°$

$d = \frac{a}{2\sin\frac{\alpha}{2}} = \frac{6}{2\sin 10°} \simeq 17{,}28$

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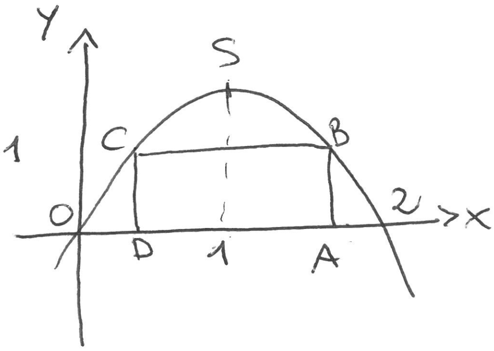
3) Die Parabel $y = -x^2 + 2x$ hat Nullstellen 0 und 2 und ist symmetrisch bezüglich $x = 1$



Sei $A = (x_A, 0)$ mit $x_A \in [1,2]$

Dann

$\rightarrow B = (x_A, -x_A^2 + 2x_A)$

$C = (2 - x_A, -x_A^2 + 2x_A)$

$D = (2 - x_A, 0)$

$A_{ABCD} = \overline{AD} \cdot \overline{BA} = (2x_A - 2) \cdot (-x_A^2 + 2x_A)$

$A(x) = (2x-2)(-x^2+2x) = -2x^3 + 6x^2 - 4x$

$A'(x) = -6x^2 + 12x - 4 = 0 \rightarrow x = \frac{3 \pm \sqrt{3}}{3}$

$\frac{3 + \sqrt{3}}{3} \in [1,2]$

$A = \left(\frac{3+\sqrt{3}}{3} \middle| 0\right)$ $B = \left(\frac{3+\sqrt{3}}{3} \middle| \frac{2}{3}\right)$ $C = \left(\frac{3-\sqrt{3}}{3} \middle| \frac{2}{3}\right)$ $D = \left(\frac{3-\sqrt{3}}{3} \middle| 0\right)$