Pass W 2010

<u>Musterlösungen:</u>

1.) Ansatz: $k: y(x) = A(x+2)^3 \cdot (x-1)^2 \cdot (x-4)$

$P\binom{-1}{2} \in k: \; 2 = (-1+2)^3 \cdot (-1-1)^2 \cdot (-1-4) \cdot A = -20A \rightarrow A = \frac{-1}{10}$

$y(x) = -\frac{1}{10}(x+2)^3(x-1)^2(x-4) \rightarrow y'(x) = (-3/5)(x-1)(x+2)^2(x^2-3x-1)$

$x^2 - 3x - 1 = 0 \rightarrow x = (3 \pm \sqrt{13})/2 \rightarrow \underline{\underline{H_1\binom{-0.3028}{3.5703},\ H_2\binom{3.3028}{55.1297}}}$

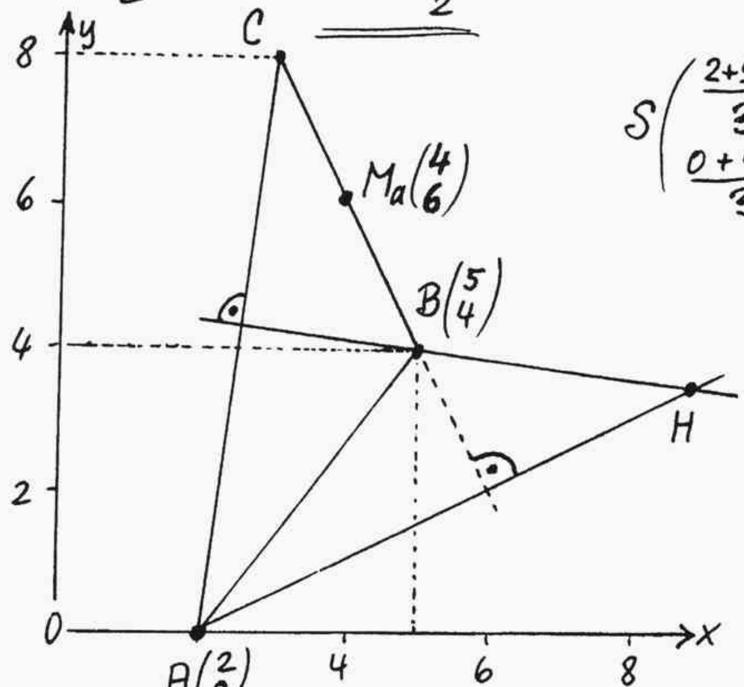
Tiefpunkt: $\underline{\underline{T\binom{1}{0}}}$

Ausserdem Terrassenpunkt $S\binom{-2}{0}$

2a) $\vec{d} + 4\vec{x} = 2\vec{a} \longrightarrow \underline{\underline{x = \frac{\vec{a}}{2} - \frac{\vec{d}}{4}}}, \; \vec{x} + \vec{y} = \vec{a} \rightarrow \vec{y} = \vec{a} - \vec{x} = \underline{\underline{-\vec{a} - \frac{\vec{d}}{2}}}$

$-\vec{z} = \vec{a} + \frac{\vec{d}}{2} \rightarrow \vec{z} = \underline{\underline{-\vec{a} - \frac{\vec{d}}{2}}}$

b)



$S\begin{pmatrix} \frac{2+5+3}{3} \\ \frac{0+4+8}{3} \end{pmatrix} \rightarrow \underline{\underline{S\begin{pmatrix} 10/3 \\ 4 \end{pmatrix}}}$

$g_1: m_1 = -\left(\frac{4-8}{5-3}\right)^{-1} = \frac{1}{2} \rightarrow A\binom{2}{0} \in g_1: 0 = \frac{2}{2} + q_1 \rightarrow q_1 = -1 \rightarrow y = \frac{x}{2} - 1$

$g_2: m_2 = \left(\frac{8-0}{3-2}\right)^{-1} = -\frac{1}{8} \rightarrow B\binom{5}{4} \in g_2: 4 = -\frac{5}{8} + q_2 \rightarrow q_2 = \frac{37}{8} \rightarrow y = -\frac{x}{8} + \frac{37}{8}$

$H = g_1 \cap g_2: \frac{x}{2} - 1 = -\frac{x}{8} + \frac{37}{8} \rightarrow \frac{5x}{8} = \frac{45}{8} \rightarrow x = 9, \; y = \frac{x}{2} - 1 = \frac{7}{2}$

$\rightarrow \underline{\underline{H\binom{9}{7/2}}}$

3.) $f: y = ax^3 + bx^2 + cx$

$y' = 3ax^2 + 2bx + c$

$y'(0) = c = 3$

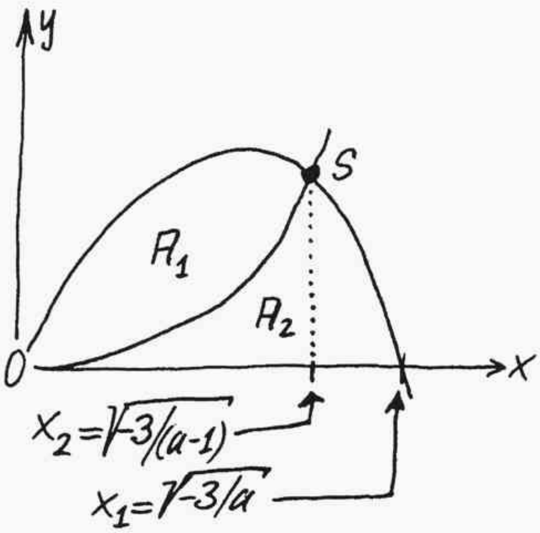
$y'' = 6ax + 2b$

$y''(0) = 2b = 0 \rightarrow b = 0$

$k: y = ax^3 + 3x$

$(ax^2 + 3)x = 0 \rightarrow x_1 = \sqrt{-3/a}$

$ax^3 + 3x = x^3 \rightarrow x_2 = \sqrt{-3/(a-1)}$



$$\int_0^{x_2} [(a-1)x^3 + 3x]\,dx = \frac{1}{2}\int_0^{x_1} [ax^3 + 3x]\,dx$$

$$\frac{(a-1)}{4}x_2^4 + \frac{3}{2}x_2^2 = \frac{1}{2}\left[\frac{a}{4}x_1^4 + \frac{3}{2}x_1^2\right]$$

$$\frac{(a-1)}{4}\cdot\frac{9}{(a-1)^2} - \frac{9}{2(a-1)} = \frac{1}{2}\left[\frac{a}{4}\cdot\frac{9}{a^2} - \frac{9}{2a}\right] \rightarrow \frac{18}{a-1} = \frac{9}{a} \rightarrow a = -1$$

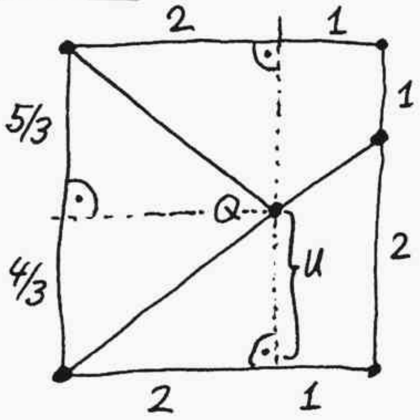
$$\underline{\underline{f: y = -x^3 + 3x}}$$

4.) Annahme: $\overline{AB} = 3$, Fläche $= 9$

$\overline{AB} \cdot \overline{BP}/2 = 3 \rightarrow \alpha = \arctan(\overline{BP}/\overline{AB})$

$= \arctan(6/(\overline{AB})^2) = \arctan(2/3)$

$\underline{\underline{\alpha = 33.69°}}$

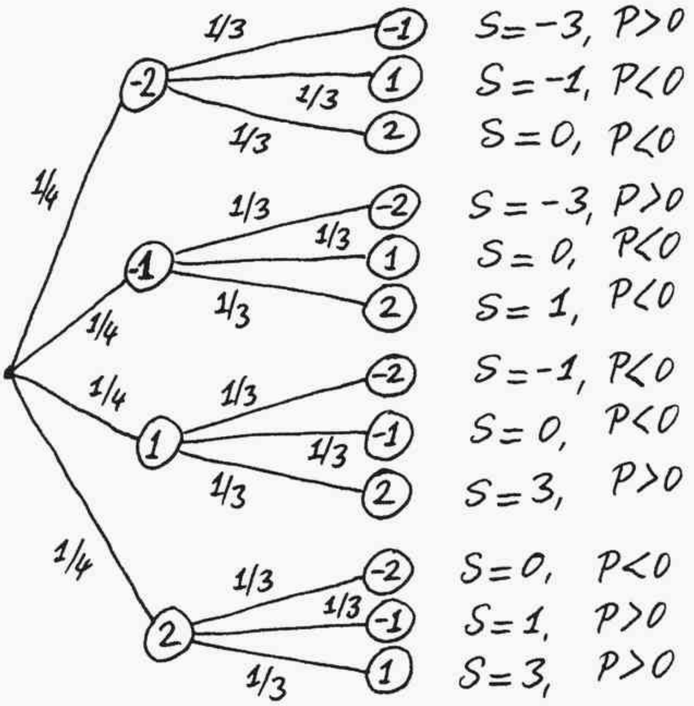


Strahlensatz: $\frac{u}{2} = \frac{2}{3} \rightarrow u = 4/3$

$\delta = \arctan((5/3)/2) = \arctan(5/6)$

$\underline{\underline{\delta = 39.81°}}$

5a)



a.1) $P(S=0) = 1/3$

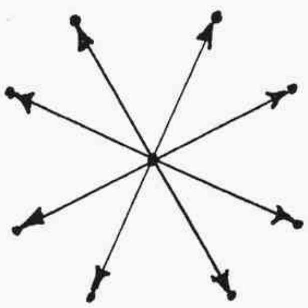
a.2) $\underline{\underline{P(P>0) = 1/3}}$

b)

b \ a	-2	-1	1	2
-2	$1/16$	$1/16$	$1/16$	$1/16$
-1	$1/16$	$1/16$	$1/16$	$1/16$
1	$1/16$	$1/16$	$1/16$	$1/16$
2	$1/16$	$1/16$	$1/16$	$1/16$

b.1) $P(\vec{v}_1 = \vec{v}_2) = \underline{\underline{1/16}}$

Gruppe A
$|\vec{v}| = \sqrt{3}$

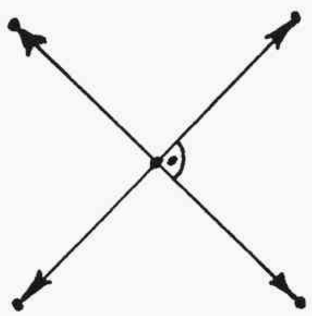


$P(A) = \frac{8}{16} = \frac{1}{2}$

Gruppe B
$|\vec{v}| = \sqrt{2}$

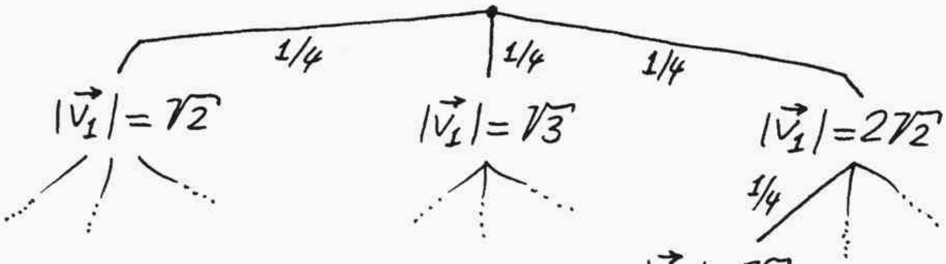
$P(B) = \frac{4}{16} = \frac{1}{4}$

Gruppe C
$|\vec{v}| = 2\sqrt{2}$



$P(C) = \frac{4}{16} = \frac{1}{4}$

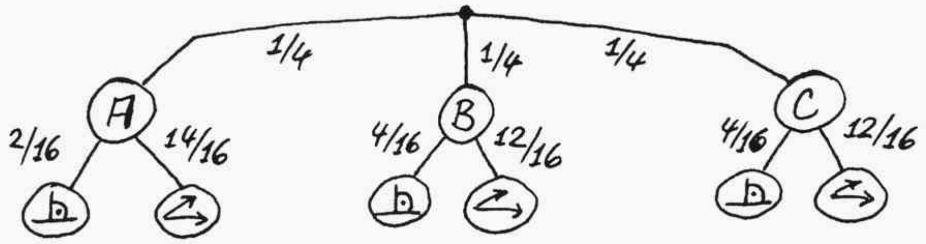
b.2)



$$\underbrace{|\vec{v}_2| = \sqrt{2}}_{|\vec{v}_1| = 2|\vec{v}_2|}$$

$$P(|\vec{v}_1| = 2|\vec{v}_2|) = \frac{1}{4}\cdot\frac{1}{4} = \underline{\underline{\frac{1}{16}}}$$

b.3)



$$P = \frac{1}{2}\cdot\frac{1}{8} + \frac{1}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4} = \underline{\underline{\frac{3}{16}}}$$

6.) $f(x) = \frac{1}{b}e^{-x}[a - x^2] \rightarrow f'(x) = \frac{1}{b}e^{-x}[x^2 - 2x - a]$

$f(-1) = \frac{e}{b}(a-1) = e \qquad f'(-1) = \frac{1}{b}\cdot e[1+2-a] = \frac{e}{b}(3-a) = 0$

$a = 3 \rightarrow \frac{e}{b}(3-1) = \frac{2e}{b} = e \rightarrow b = 2 \rightarrow \underline{\underline{f(x) = \frac{1}{2}[3-x^2]e^{-x}}}$

Nullstellen:

$\frac{1}{2}\underbrace{[3-x^2]}\underbrace{e^{-x}}_{>0} = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$x_1 = -\sqrt{3}$

$\underline{\underline{x_2 = \sqrt{3}}}$