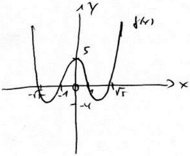
Pass F13

1. a) $f(x) = x^4 - 6x^2 + 5$ Achsensymmetrie, nur gerade Exp.

$f'(x) = 4x^3 - 12x$

$f''(x) = 12x^2 - 12$



NST: $(x^2-5)(x^2-1) = 0$

$x_{1,2} = \pm\sqrt{5}$

$x_{3,4} = \pm 1$

Extr.: $4x(x^2-3) = 0$

$x_1 = 0$ $\quad y_1 = 5$ $\quad f''(0) < 0 \Rightarrow Max(0|5)$

$x_{2,3} = \pm\sqrt{3}$ $\quad y_{2,3} = -4$ $\quad f''(\pm\sqrt{3}) > 0 \Rightarrow Min(\pm\sqrt{3}|-4)$

WP $12(x-1)(x+1) = 0$

$x_{1,2} = \pm 1$ $\quad y_{1,2} = 0$ einfache NST, Vorzeichenwechsel, $WP(\pm 1, 0)$

Wertebereich: $W = [-4; \infty[$ wegen Minima und Achsensymmetrie

b) $g(x) = x^2 + 5$ $\qquad f = g$

$g'(x) = 2x$ $\qquad x^2(x^2-7) = 0$

$x = 0$

$x = \pm\sqrt{7}$

$\tan\alpha_f = f'(\sqrt{7}) = 16\sqrt{7}$

$\alpha_f = 88{,}64°$

$\tan\alpha_g = g'(\sqrt{7}) = 2\sqrt{7}$

$\alpha_g = 79{,}298°$

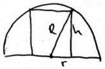
$\underline{\alpha} = \alpha_f - \alpha_g = \underline{9{,}35°}$

c) $A = \int_{-\sqrt{7}}^{\sqrt{7}} (g-f)\,dx = \underbrace{2\int_0^{\sqrt{7}} (g-f)\,dx}_{\text{Symmetrie}} = 2 \cdot \int_0^{\sqrt{7}} (-x^4 + 7x^2)\,dx$

$= 2\left[-\frac{1}{5}x^5 + \frac{7}{3}x^3\right]_0^{\sqrt{7}} = \underline{\frac{196}{15}\sqrt{7}}$

2.

Pass F13



$R^2 = h^2 + r^2$

$r^2 = R^2 - h^2$

$V = r^2 \pi h = (R^2 - h^2)\pi h = \pi(R^2 h - h^3)$ $\qquad D_h = [0; R]$

$V' = \pi(R^2 - 3h^2) = 0$

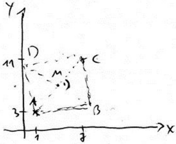
$h = (\pm) \frac{1}{3}\sqrt{3} R$

Ränder D: $V(0) = 0$

$V(R) = 0$

$V(h) = \frac{2}{9}\pi\sqrt{3} r^3$ $\qquad$ 58% Halbkugelvolumen

3. Pass F13



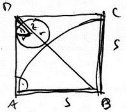
$$\vec r_M = \frac{\binom{1}{3}+\binom{7}{11}}{2} = \binom{4}{7} \qquad M(4|7)$$

$$\vec{AC} = \binom{6}{8} \qquad \vec{AM} = \tfrac12 \vec{AC} = \binom{3}{4} \xrightarrow{D} \vec{MB} = \binom{4}{-3}$$

$$\vec{MD} = \binom{-4}{3}$$

$$\vec r_B = \vec r_M + \vec{MB} = \binom{4}{7}+\binom{4}{-3} = \binom{8}{4} \qquad \underline{B(8|4)}$$

$$\vec r_D = \vec r_M + \vec{MD} = \binom{4}{7}+\binom{-4}{3} = \binom{0}{10} \qquad \underline{D(0|10)}$$



$$\frac{DZ}{r} = \frac{DB}{s} \qquad DZ = s\cdot\sqrt2 - s - r$$

$$DB = s\sqrt2$$

$$\frac{s\sqrt2 - s - r}{r} = \frac{s\sqrt2}{s}$$

$$s(\sqrt2-1) = r(1+\sqrt2)$$

$$r = \frac{\sqrt2-1}{\sqrt2+1}\,s$$

$$\underline{r = (3-2\sqrt2)\,s}$$

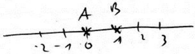
$$\vec{DZ} = r \qquad \vec{BZ} = \frac{s+r}{s\sqrt2}\cdot\vec{BD} = \frac{(4-2\sqrt2)s}{s\sqrt2}\cdot\binom{-8}{6} = (2\sqrt2-2)\binom{-8}{6}$$

$$\vec r_Z = \vec r_B + \vec{BZ} = \binom{8}{4} + (2\sqrt2-2)\binom{-8}{6} = 4\binom{6-4\sqrt2}{-2+3\sqrt2} \approx \binom{5{,}5}{3{,}6}$$

$$\underline{Z(24-16\sqrt2 \,|\, -8+12\sqrt2)}$$

Pass F13

4.



A, B pro Sekunde ±1, $p = 50\%$

a) P(A nach einer Sekunde rechts von B) : A nach rechts, B nach links

$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 25\%$

b) P(" zwei " ") :

je $\left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{16}$

A → 2 : 1 } 3
B → 1, 0, −1 3
2 − 1

A → 1 : −

A → 0 : 2 } 2
B → −1, 1

A → −1, −2 ↯

5

$= 5 \cdot \frac{1}{16} = \frac{5}{16} = 31{,}3\%$

c) P(A, B nach 10 Sek. am gleichen Platz)

d.h. jeder 5 mal nach rechts, 5 mal nach links $\binom{10}{5} = 252$

$= 252 \cdot \left(\frac{1}{2}\right)^{10} \cdot 252 \cdot \left(\frac{1}{2}\right)^{10} = 6{,}1\%$

5. $f(x) = 2e^{-x} - 1$

Pass F13

a) $f(0) = \underline{1 = y}$

$2e^{-x} - 1 = 0$

$e^{-x} = \frac{1}{2}$

$\underline{x} = -\ln\frac{1}{2} = \underline{\ln 2}$

$f'(x) = -2e^{-x}$

$f'(\ln 2) = -2e^{-\ln 2} = -1 = \tan\alpha \Rightarrow \underline{\alpha = -45° = 135°}$

$A = \int_0^{\ln 2} f(x)\,dx = \left[-2e^{-x} - x\right]_0^{\ln 2} = -2e^{-\ln 2} - \ln 2 + 2$

$\underline{A = 1 - \ln 2}$

b) $g(x) = ae^{-x} - 1$ $\qquad e^{-x} = \frac{1}{a}$

$g'(x) = -ae^{-x}$ $\qquad x = \ln a$

$g'(\ln a) = -ae^{-\ln a} = \frac{-a}{e^{\ln a}} = \frac{-a}{a} = -1 = \tan\alpha$

$\underline{\alpha = -45° = 135°}$

g^{-1}: $x = a \cdot e^{-y} - 1$

$e^{-y} = \frac{1+x}{a}$

g^{-1}: $y = -\ln\frac{1+x}{a} = \ln\left(\frac{a}{1+x}\right)$

$g^{-1}(0) = 2 = \ln\left(\frac{a}{1+0}\right) = \ln a$

$\underline{e^2 = a}$

oder: Umkehrfunktion durch (0|2) bedeutet (2|0) auf Funktion:

$g(2) = 0 = ae^{-2} - 1$

$\underline{e^2 = a}$