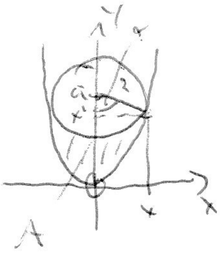
Pass F17

1.



Steigung in $(x|x^2)$: $2x$

: $\frac{a-x^2}{x}$

$a-x^2$

$\underbrace{2x}_{>0}\cdot\underbrace{\frac{a-x^2}{x}}_{>0}=1$

$a-x^2=\frac{1}{2}$

Kreis: $(a-x^2)^2+x^2=2^2$

$x^2=\frac{15}{4}$

$x=\frac{1}{2}\sqrt{15}$ $\longrightarrow$ $a=\frac{17}{4}$

$A=2\cdot(A_{Trapez}-A_{Sektor}-A_{Parabel})$

Symmetrie

$=2\cdot\left(\frac{a+x^2}{2}\cdot x-\frac{\alpha}{360}\cdot r^2\pi-\int_0^x x^2\,dx\right)$

$=2\cdot\left(\frac{17/4+15/4}{2}\cdot\frac{1}{2}\sqrt{15}-\frac{75{,}52}{360}\cdot 4\pi-\left[\frac{1}{3}x^3\right]_0^{\frac{1}{2}\sqrt{15}}\right)$

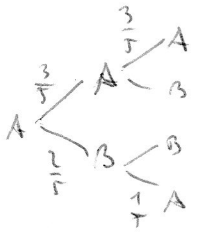
$=5{,}4$

$\tan\alpha=\frac{x}{a-x^2}=\frac{\frac{1}{2}\sqrt{15}}{\frac{1}{2}}$

$\alpha=75{,}52°$

2. $p_{AB}=\frac{2}{5}$ $p_{BA}=\frac{1}{5}$ $p_{AA}=\frac{3}{5}$ $p_{BB}=\frac{4}{5}$ $t=0$: A

a)



$P(A \text{ nach 2 Sekunden})=\left(\frac{3}{5}\right)^2+\frac{2}{5}\cdot\frac{1}{5}=\frac{11}{25}=44\%$

b) $P(\text{mind. einmal in } B)>99\%$

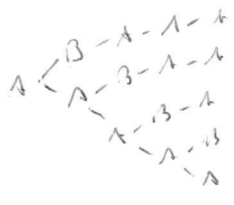
$1-P(\text{nie in } B)>0{,}99$

$\left(\frac{3}{5}\right)^n<0{,}01$

$n>\log_{3/5}0{,}01=9{,}01$

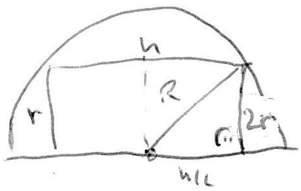
Nach 10 Sekunden

c)



In AAAB wechselt er nur auf B, aber nicht wieder zurück zu A: (3/5)^3*2/5
In den anderen 3 Fällen wechselt er einmal von A zu B: 2/5, einmal von B zu A: 1/5 und zweimal bleibt er auf A: 3/5
also: 2/5*1/5*(3/5)^2 und das in 3 Ästen
ingesamt: 17,3%

3.



$R^2=\left(\frac{h}{2}\right)^2+(2r)^2$

$r^2=\frac{1}{4}R^2-\frac{1}{16}h^2$

$=\frac{1}{16}(4R^2-h^2)$

$V=r^2\pi\cdot h\to\max$

$V=\frac{1}{16}(4R^2-h^2)\pi h$ $\quad h\in[0;2R]$

$=\frac{\pi}{16}(-h^3+4R^2h)$

$V'=\frac{\pi}{16}(-3h^2+4R^2)$

$V''=\frac{\pi}{16}(-6h)$

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3. $V' = 0$

$h = \frac{+2}{(-)3}\sqrt{3}R \qquad r = \frac{1}{6}\sqrt{6}R$

$V''(h) < 0 \Rightarrow Max$

$\frac{V_t}{V_{HK}} = \frac{\pi r^2 h}{\frac{2}{3}\pi R^3} = \frac{\frac{1}{6}R^2 \cdot \frac{2}{3}\sqrt{3}R}{\frac{2}{3}R^3} = \frac{1}{6}\sqrt{3} = 28{,}9\ \%$

Ränder D: $\left.\begin{array}{l} V(0) = 0 \\ V(R) = 0 \end{array}\right\}$ Min

4. $g_1: 3x - ay + a^2 + 4a = 0 \qquad g_2: (a-7)x + 4y - 8a = 0 \qquad a \neq 0$

a) $a = 1$

$3x - y + 5 = 0$
$-6x + 4y - 8 = 0$

$\underline{S(-2|-1)}$

b)

$D = \begin{vmatrix} 3 & -a \\ a-7 & 4 \end{vmatrix} = 12 + a(a-7) = 0$

$a = 3$
$a = 4$

$a = 3: \quad D_1 = \begin{vmatrix} -21 & -3 \\ 24 & 4 \end{vmatrix} \neq 0$ parallel

$D_2 = \begin{vmatrix} 3 & -21 \\ -4 & 24 \end{vmatrix} \neq 0$

$a = 4 \quad D_1 = \begin{vmatrix} -32 & -4 \\ 32 & 4 \end{vmatrix} = 0$ identisch

$D_2 = \begin{vmatrix} 3 & -32 \\ -3 & 32 \end{vmatrix} = 0$

c) $\vec{n}_1 \cdot \vec{n}_2 = 0$

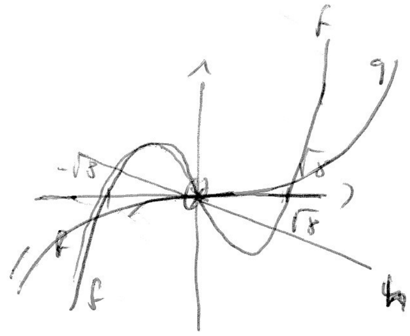
$\begin{pmatrix} 3 \\ -a \end{pmatrix}\begin{pmatrix} a-7 \\ 4 \end{pmatrix} = 0$

$3(a-7) - 4a = 0$

$\underline{a = -21}$

5. $f(x) = x^3 - 8x \qquad g(x) = \frac{1}{9}x^3$

$= x(x-\sqrt{8})(x+\sqrt{8})$



$f = g$

$\frac{8}{9}x^3 - 8x = 0$

$8x\left(\frac{1}{3}x - 1\right)\left(\frac{1}{3}x + 1\right) = 0$

$x = 0 \qquad x = \pm 3$

a) $\underline{A} = \int_0^3 (g - f)\,dx = \left[4x^2 - \frac{2}{9}x^4\right]_0^3 = \underline{18}$

b) $y = mx \qquad m \in \mathbb{Z}^-$

$x^3 - 8x = mx$

$x(x^2 - 8 - m) = 0$

$x = 0 \qquad x = \pm\sqrt{8+m}$

$9 = \int_0^{\sqrt{8+m}} (h - f)\,dx$

$9 = \left[(m+8)\frac{x^2}{2} - \frac{1}{4}x^4\right]_0^{\sqrt{8+m}}$

$9 = \frac{1}{2}\underbrace{(m+8)}^2 - \frac{1}{4}(m+8)^2$

$9 = \frac{1}{4}(m+8)^2$

$\underline{m_1 = -2}$
$(m_2 = -14)$