Lösungen Eidg. Prüfung Passerelle Winter 2023

① a) $P = \frac{3}{9} = \underline{\underline{\frac{1}{3}}}$

b) $P = \frac{6}{9} \cdot \frac{5}{8} = \frac{10}{24} = \underline{\underline{\frac{5}{12}}}$

c) $P =$ P(2 gerade Zahlen) + P(2 ungerade Zahlen) =
$= \frac{6}{9} \cdot \frac{5}{8} + \frac{3}{9} \cdot \frac{2}{8} = \frac{36}{72} = \underline{\underline{\frac{1}{2}}}$

d) $P = 1 -$ P(Produkt ungerade) $= 1 - \frac{3 \cdot 2}{9 \cdot 8} = 1 - \frac{1}{12} = \underline{\underline{\frac{11}{12}}}$

e) $P = P(2/3) + P(2/4) + P(3/4) = \frac{2 \cdot 3}{9 \cdot 8} + \frac{2 \cdot 4}{9 \cdot 8} + \frac{3 \cdot 4}{9 \cdot 8} = \underline{\underline{\frac{13}{36}}}$

f) $P = P(4/3) + P(4/2) + P(3/2) = \underline{\underline{\frac{13}{36}}}$

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② a) $D = \mathbb{R}$; weder symmetrisch zur y-Achse, noch zum Nullpunkt.

Nullstellen: $f(x) = 0 \rightarrow x \cdot (x^2 - x - 6) = 0 \rightarrow x \cdot (x-3) \cdot (x+2) = 0$
$\rightarrow \underline{\underline{N(0/0)}}, \underline{\underline{N(3/0)}}, \underline{\underline{N(-2/0)}}$

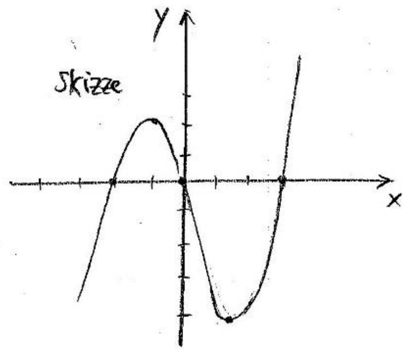
$f'(x) = \frac{3}{2}x^2 - x - 3, \quad f''(x) = 3x - 1, \quad f'''(x) = 3$

setze $f'(x) = 0 \rightarrow 3x^2 - 2x - 6 = 0 \rightarrow x = \frac{2 \pm \sqrt{76}}{6} = \begin{cases} 1.786 \\ -1.120 \end{cases}$

$f''(1.786) > 0 \rightarrow \underline{\underline{T(1.786/-4.104)}}$
$f''(-1.120) < 0 \rightarrow \underline{\underline{H(-1.120/2.030)}}$

setze $f''(x) = 0 \rightarrow x = \frac{1}{3}, \; f'''(\frac{1}{3}) \neq 0$
$\rightarrow \underline{\underline{W(0.333/-1.037)}}$

Skizze



Wertemenge $W = \mathbb{R}$

b) $\int_{-2}^{0} f(x)\,dx = \left[\frac{1}{8}x^4 - \frac{1}{6}x^3 - \frac{3}{2}x^2\right]_{-2}^{0} = \underline{\underline{\frac{8}{3} = A_1}}$

$\int_{0}^{3} f(x)\,dx = -\frac{63}{8} \quad \rightarrow \underline{\underline{A_2 = \frac{63}{8}}}$

c) $\int_{0}^{\pi} k \cdot \sin x\,dx = \left[-k \cdot \cos x\right]_0^{\pi} = k + k = 2k \quad \rightarrow \underline{\underline{k = \frac{63}{16}}}$

③ a) Punkt B in E einsetzen: $0-0=0$ erfüllt → B liegt in E.

$h: \vec{r} = \begin{pmatrix} 8 \\ 4 \\ -3 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$;

Schnittpunkt $C = h \cap E$: $8+t-4+t=0 \rightarrow t=-2 \rightarrow \underline{\underline{C(6/6/-3)}}$

b) $\vec{AB} = \begin{pmatrix} -8 \\ -4 \\ 8 \end{pmatrix}$, $\vec{AC} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$, $\vec{BC} = \begin{pmatrix} 6 \\ 6 \\ -8 \end{pmatrix}$

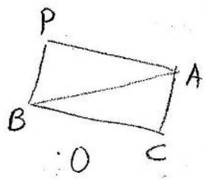
$\vec{AC} \cdot \vec{BC} = 0 \rightarrow$ rechter Winkel bei C.

$|\vec{AC}| = \sqrt{8}$, $|\vec{BC}| = \sqrt{136}$; Flächeninhalt $A = \frac{1}{2} \cdot \sqrt{8} \cdot \sqrt{136} =$

$= \frac{1}{2} \cdot \sqrt{8} \cdot \sqrt{8} \cdot \sqrt{17} = \underline{\underline{4\sqrt{17}}}$

c) $\vec{OP} = \vec{OB} + \vec{CA} =$

$= \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} \rightarrow \underline{P(2/-2/5)}$



$\vec{PC} = \begin{pmatrix} 4 \\ 8 \\ -8 \end{pmatrix}$

$\cos \phi' = \frac{\vec{PC} \cdot \vec{AB}}{|\vec{PC}| \cdot |\vec{AB}|} = \frac{-32-32-64}{\sqrt{144} \cdot \sqrt{144}} = -0,\bar{8} \rightarrow \phi' = 152,734°$

$\phi = 180° - \phi' = \underline{\underline{27,266°}}$

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④ 4.1 $\sin x + \frac{1}{2} \cdot \frac{\sin x}{\cos x} = 0 \quad | \cdot \cos x$

$\sin x \cdot \cos x + \frac{1}{2} \cdot \sin x = 0$

$\sin x \cdot (\cos x + \frac{1}{2}) = 0$

$\sin x = 0 \rightarrow x = \underline{\underline{0, \pi, 2\pi}}$; $\cos x = -\frac{1}{2} \rightarrow x = \underline{\underline{\frac{2\pi}{3}, \frac{4\pi}{3}}}$

4.2 $K_1: \quad x^2 + (y-6)^2 = 100$ subtrahieren

$K_2: \quad (x-10,5)^2 + (y-6)^2 = 42,25$

$x^2 - (x^2 - 21x + 110,25) = 57,75 \rightarrow 21x = 168 \rightarrow x = 8$

in K_1: $64 + (y-6)^2 = 100 \rightarrow (y-6)^2 = 36 \rightarrow y-6 = \pm 6$

$\rightarrow y = 0$ oder $y = 12$

$\Rightarrow \underline{\underline{S_1(8/0)}}, \underline{\underline{S_2(8/12)}}$

4.3 $\frac{x-1}{1-2x} - x^2 = 2x \cdot (x^2+1) + \frac{x}{2x-1} \quad | \cdot (1-2x)$

$x-1 - (1-2x) \cdot x^2 = 2x \cdot (x^2+1) \cdot (1-2x) - x$

$\cancel{2x^3} - x^2 + \cancel{x} - 1 = -4x^4 + \cancel{2x^3} - 4x^2 + \cancel{x}$

$4x^4 + 3x^2 - 1 = 0$; $x^2 = u \rightarrow 4u^2 + 3u - 1 = 0$

$u = \frac{-3 \pm 5}{8} = \begin{cases} -1 \\ 1/4 \end{cases}$

$u = -1 \rightarrow$ kein x-Wert

$u = 1/4 \rightarrow x = 1/2, \; x = -1/2$

$D = \mathbb{R} \setminus \{\frac{1}{2}\} \rightarrow \underline{\underline{L = \{-\frac{1}{2}\}}}$