Lösungen zu Passerellen-Prüfungen He 2006

1. $y = ax^3 + bx^2 + cx + d$

Ursprung: $d = 0$

Tangente: $y' = 3ax^2 + 2bx + c \Rightarrow c = 0$

mit $y = 6x - 1 \overset{m=6}{\Rightarrow} \boxed{6 = 27a - 6b}$

$y = ax^3 + bx^2 = x^2(ax + b)$ $\quad B(-3/0)$ einsetzen

$\boxed{0 = 9(-3a + b)}$

$\underline{a = \frac{2}{3}\,;\ b = 2}$

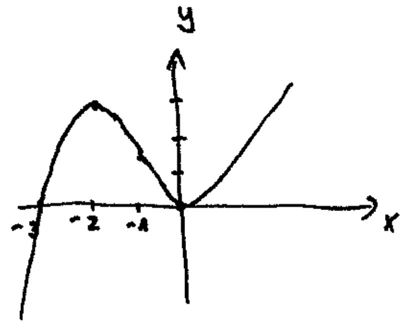
$f(x) = \frac{2}{3}x^3 + 2x^2$

NS: $(0/0)\,;\ (-3/0)$

$f'(x) = 2x^2 + 4x = 0 \Rightarrow \underline{Min(0/0)\,;\ Max(-2/\tfrac{8}{3})}$

$f''(x) = 4x + 4 = 0 \Rightarrow \underline{WP(-1/\tfrac{4}{3})}$

$f'''(x) = 4 \neq 0$



2. $V = 2m^3$ $\quad$ Materialkosten minimal: $K(a,h) = a^2 + 4\cdot a\cdot h\cdot 2$ $\quad V = a^2\cdot h \Rightarrow h = \frac{V}{a^2}$

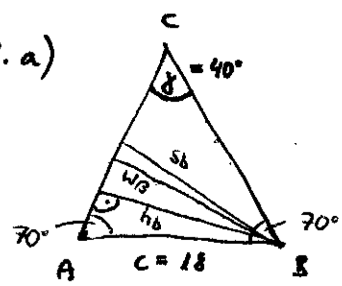
$$K(a) = a^2 + \frac{8V}{a}$$

$$K'(a) = 2a - \frac{8V}{a^2} \overset{!}{=} 0 \Rightarrow a = \sqrt[3]{4V} = \sqrt[3]{8} = 2$$

$$K''(a) = 2 + \frac{16V}{a^3} > 0 \longrightarrow \text{Minimum}$$

$\left.\begin{array}{l} a = 2m \\ b = h = \frac{1}{2}m \end{array}\right\}$

3. a)



$\underline{h_b} = 18\cdot\sin 70° = \underline{16.9}$

$\frac{h_b}{w_\beta} = \cos 15° \Rightarrow \underline{w_\beta = 17.5}$

$\underline{a = b} = \frac{h_b}{\cos 50°} = \underline{26.3}$

$\underline{s_b} = \sqrt{18^2 + \left(\frac{26.3}{2}\right)^2 - 2\cdot 18\cdot\frac{26.3}{2}\cdot\cos 70°} = \underline{18.3}$

b) $\tan(45° + \beta) - \tan(45° - \beta) = \frac{\tan 45° + \tan\beta}{1 - \tan 45°\cdot\tan\beta} - \frac{\tan 45° - \tan\beta}{1 + \tan 45°\tan\beta}$

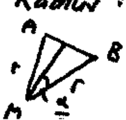
$$= \frac{(1 + \tan\beta)^2 - (1 - \tan\beta)^2}{1 - \tan^2\beta} = \frac{4\tan\beta}{1 - \tan^2\beta} = 2\,\frac{2\tan\beta}{1 - \tan^2\beta} = \underline{2\tan(2\beta)}$$

4. Gerade (AB): $m = \frac{\Delta y}{\Delta x} = \frac{3}{9} = \frac{1}{3} \Rightarrow m_{\perp} = -3$

Mittelpunkt von $\overline{AB}$: $x = \frac{12+21}{2} = 16.5$ $\quad y = \frac{13+16}{2} = 14.5$ $\quad M_{AB}(16.5/14.5) \Rightarrow \underline{y = -3x + 64}$

[illegible]: $-\frac{2}{3}x + \frac{80}{3} = -3x + 64 \Rightarrow x = 16 \Rightarrow y = 16$ $\quad \underline{M(16/16)}$

Radius: $\overline{MA} = r = 5$



$\overline{AB} = \sqrt{50}$ $\quad \frac{\alpha}{2} \Rightarrow \alpha = 143.13° \Rightarrow A = \frac{r^2\cdot\pi\cdot\alpha}{360°} \simeq \underline{31.2}$

5. a) $P = \underbrace{\frac{5}{9}\cdot\frac{4}{8}\cdot\frac{3}{7}\cdot\frac{2}{6}}_{\text{alles Treffer}} + \underbrace{4\left(\frac{5}{9}\cdot\frac{4}{8}\cdot\frac{3}{7}\cdot\frac{4}{6}\right)}_{\text{1 Niete}} = \frac{5}{14} = \underline{35.7\%}$

b) $P = \frac{3}{7}\cdot\frac{2}{6} + \frac{3}{7}\cdot\underset{\uparrow \atop \text{Niete}}{\frac{4}{6}} + \underset{\uparrow \atop \text{Niete}}{\frac{4}{7}}\cdot\frac{3}{6} = \frac{5}{7} = \underline{71.4\%}$

c) $1 - P(\text{kein Gewinn}) - P(1\ \text{Gewinn}) - P(2\ \text{Gewinne})$

$$1-\left(\frac{9}{14}\right)^8 - 8\cdot\left(\frac{9}{14}\right)^7\left(\frac{5}{14}\right) - 28\left(\frac{9}{14}\right)^6\left(\frac{5}{14}\right)^2 = 1-\left(\frac{9}{14}\right)^6\left[\left(\frac{9}{14}\right)^2+8\left(\frac{9}{14}\right)\left(\frac{5}{14}\right)+28\left(\frac{5}{14}\right)^2\right]$$

$$= 1-\left(\frac{9}{14}\right)^6\left(\frac{1141}{196}\right)$$

$$= \underline{58.9\%}$$

N N G G G G G G   7x
G N N ...   6x
G G N N ...   5x
G G G N ...   4x
G G G G N ..   3x
⋮   2x

28x

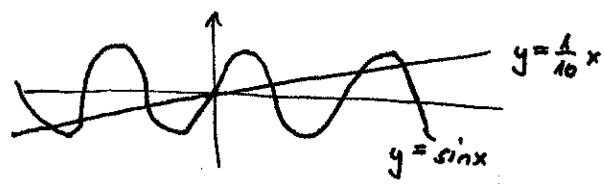
6.1. $a - b = 1$
$\ln a - \ln b = 1$

$\ln a - \ln(a-1) = 1$

$\ln\left(\frac{a}{a-1}\right) = 1$

$\frac{a}{a-1} = e \Rightarrow \underline{a = \frac{e}{e-1}}$ $\qquad \underline{b = a-1 = \frac{1}{e-1}}$

6.2. Graphisch $\Rightarrow$ $\underline{7\ \text{Lösungen}}$



6.3. Annahme: $a_n = 9^n - 1$ durch 8 teilbar

zu zeigen: $a_{k+1} = 9^{n+1} - 1$ ist durch 8 teilbar

Beweis: $k=1$: 8 ist durch 8 teilbar

$$a_{k+1} = 9^{n+1} - 1 = \underbrace{9(9^n - 1)}_{\text{ist durch 8 teilbar}} + 8$$

q.e.d.