Pass #13

1. $P(A \text{ gewinnt}) = w + sssw + ssssssw$

$= \frac{4}{12} + \frac{8 \cdot 7 \cdot 6 \cdot 4}{12 \cdot 11 \cdot 10 \cdot 9} + \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 4}{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}$

$= \frac{281}{495} = 56{,}8\,\%$

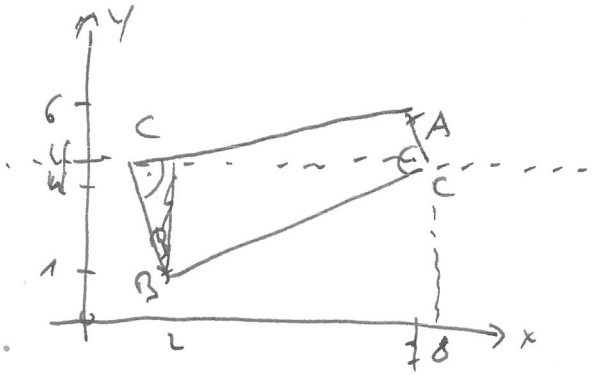
$P(B \text{ gewinnt}) = sw + ssssw + sssssssw$

$= \frac{8 \cdot 4}{12 \cdot 11} + \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8} + \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 4 \cdot 4}{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}$

$= \frac{53}{165} = 32{,}1\,\%$

$P(C) = 1 - P(A) - P(B) = \frac{1}{9} = 11{,}1\,\%$

2.



a) $\overline{CA} \cdot \overline{CB} = 0$

$\begin{pmatrix} 7-x \\ 6-4 \end{pmatrix} \begin{pmatrix} 2-x \\ 1-4 \end{pmatrix} = 0$

$(7-x)(2-x) + 2 \cdot (-3) = 0$

$x_1 = 1$

$x_2 = 8$

c) $A_\Delta = \frac{r}{2}(a+b+c) = \frac{1}{2}ab$

$r = \frac{ab}{a+b+c}$

$A_0 = r^2\pi = \left(\frac{ab}{a+b+c}\right)^2 \pi$

$a = BC = \sqrt{41}$

$b = CA = \sqrt{5}$

$b = AB = 5\sqrt{2}$

$A_0 = \frac{5}{2}(13 - 4\sqrt{10})\,\pi \approx 2{,}76$

b) ABC ist rechtwinklig, Umkreis ist Thaleskreis, Mitte bei Mitte AB, Radius $\frac{AB}{2}$

$M(AB) = (4{,}5 \mid 3{,}5)$ $\quad AB = \sqrt{5^2 + 5^2} = 5\sqrt{2}$

$R = \frac{5}{2}\sqrt{2}$

$k: \; (x - 4{,}5)^2 + (y - 3{,}5)^2 = \frac{25}{2}$

x-Achse: $y = 0$:

$(x - \frac{9}{2})^2 + (\frac{7}{2})^2 = \frac{25}{2}$

$(x - \frac{9}{2})^2 = \frac{1}{4}$

$x = \pm\frac{1}{2} + \frac{9}{2}$

$x_1 = 5$ $\quad x_2 = 4$

PassH13

3. $f(x) = (x+k)e^x$

$x = -3$ w P

a) $f'(x) = 1 \cdot e^x + (x+k)e^x = (x+k+1)e^x$

$f''(x) = 1 \cdot e^x + (x+k+1)e^x = (x+k+2)e^x$
$f'''(x) = 1 e^x + (x+k+2)e^x = (x+k+3)e^x$
$\vdots \quad \vdots$
$f^{(n)}(x) = (x+k+n)e^x$

$f''(-3) = 0 = (k-1)e^x$

$\underline{k = 1}$

b) $f(x) = (x+1)e^x$

$f'(x) = (x+2)e^x$

$f''(x) = (x+3)e^x$

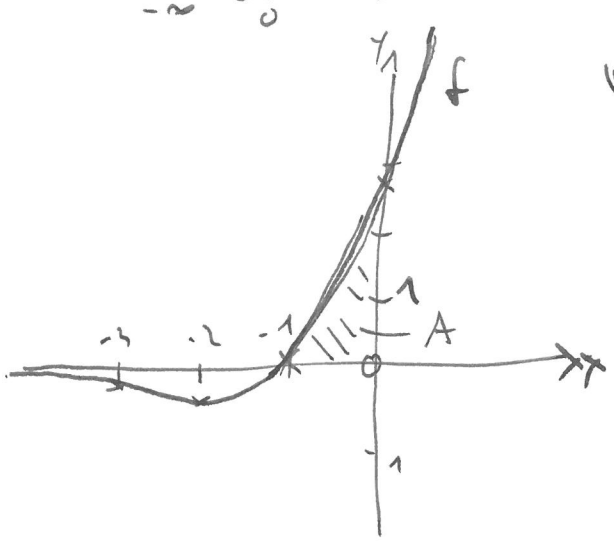
NST: $\underline{x = -1}$

Ext: $x = -2$; $y = \frac{-1}{e^2}$ $\quad f''(-2) > 0 \Rightarrow \underline{\text{Min}\left(-2 \mid \frac{-1}{e^2}\right)}$

wP: $x = -3$, einfach, $V \neq w$, $\underline{wP\left(-3 \mid \frac{-2}{e^3}\right)}$

$\lim\limits_{x \to +\infty} \underbrace{(x+1)}_{\infty} \underbrace{e^x}_{\infty} = \infty$

$\lim\limits_{x \to -\infty} \underbrace{(x+1)}_{-\infty} \underbrace{e^{-x}}_{0} = 0$ | e-Flt. stärk als Pot.



$W = \left[-\frac{1}{e^2}; \infty\right[$

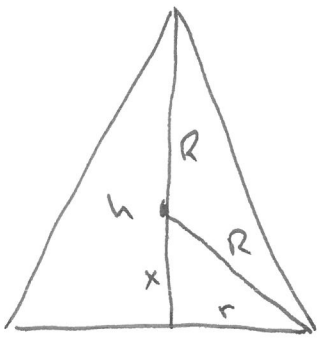
c) $A = \int_{-1}^{0} f(x)\,dx$

$= \left[x e^x\right]_{-1}^{0}$

$\underline{A = \frac{1}{e}}$

PassH13

4. $V = \frac{\pi}{3} r^2 h \to \max$



$h = R + x$

$r = \sqrt{R^2 - x^2}$

$$V = \frac{\pi}{3}(R^2 - x^2)(R+x) = \frac{\pi}{3}(-x^3 - Rx^2 + R^2x + R^3)$$

$D = [-R; R]$

$$V'(x) = \frac{\pi}{3}(-3x^2 - 2xR + R^2) = 0$$

$x_1 = -R$ $\notin D$

$x_2 = \frac{1}{3}R$

$$V''(x) = \frac{\pi}{3}(-6x - 2r)$$

$V''(-R) = 4R > 0 \Rightarrow$ Min

$V''(\frac{R}{3}) = -4R < 0 \Rightarrow$ Max $\quad V(\frac{R}{3}) = \frac{32}{81}\pi R^3 \approx 30\% V_0$

Rand: $V(-R) = 0$

$V(R) = 0$

Pass H13

5. a) $y > 0 \wedge y \neq 3$ $\quad \mathbb{D}_y = \mathbb{R}^+ \setminus \{3\}$

$x > -\frac{1}{2}$ $\quad \mathbb{D}_x =]-\frac{1}{2}; \infty[$

I. $\lg\left(\frac{y}{2x+1}\right) = 2$

$\frac{y}{2x+1} < 100$

$$\left.\begin{array}{l} \text{I.} \quad y = 200x + 100 \\ \text{II.} \quad x - 2 = 10y - 30 \end{array}\right\} \quad x = -\frac{972}{1999} \wedge y = \frac{5990}{1999}$$

b) $\frac{1}{3^x} < \frac{1}{e^{1-x}}$

$e^{1-x} < 3^x \quad | \lg$

$(1-x)\lg e < x \lg 3$

$\lg e < x \lg e + x \lg 3$

$x > \frac{\lg e}{\lg e + \lg 3} = \frac{\lg e}{\lg 3e} = \frac{1}{\ln 3 + 1}$

c) $\left.\begin{array}{l} 4\sin x = y \\ 2\tan x = y \end{array}\right\}$ $\quad 4\sin x = 2\tan x$

$2\sin x - \frac{\sin x}{\cos x} = 0$

$\sin x\left(\frac{2\cos x - 1}{\cos x}\right) = 0$

$x = 0 + 2\cdot\pi \quad | \quad x = \frac{\pi}{3} + 2\cdot\pi$

$= \frac{5}{3}\pi + 2\cdot\pi$

$\mathbb{L} = \{(0|0); (\pi|0); (\frac{\pi}{3}|2\sqrt{3}); (\frac{5}{3}\pi|-2\sqrt{3})\}$