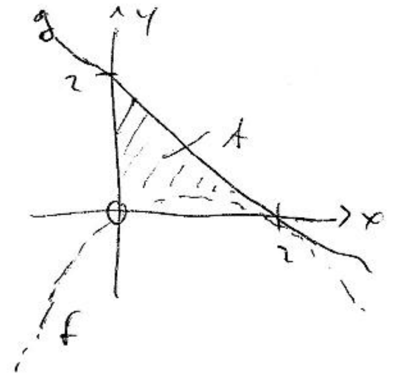
Pass H14

1. a) $g: \; y = -x + 2$ $\qquad g'(x) = -1$

NST: $x = 2$



$f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

I. $f(0) = \underline{k = c}$

II. $f(2) = g(2) = 0$

III. $f'(2) = g'(2)$

II. $4a + 2b + k = 0$

III. $4a + b = -1$

$b + k = 1$

$b = 1 - k$

$\underline{a} = \frac{1}{4}(-1-b) = \frac{1}{4}(-1-1+k) = \frac{1}{4}(-2+k)$

$$\underline{f_k(x) = \frac{1}{4}(-2+k)x^2 + (1-k)x + k}$$

$$\underline{\underline{f_0(x) = -\frac{1}{2}x^2 + x}}$$

$$A = A_\Delta - \int_0^2 f_0(x)\,dx = \frac{1}{2}\cdot 2 \cdot 2 - \left[-\frac{1}{6}x^3 + \frac{1}{2}x^2\right]_0^2$$

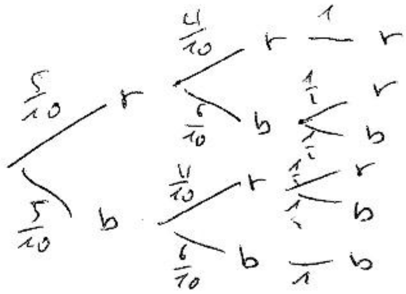
$$\underline{\underline{A}} = 2 - \frac{2}{3} = \underline{\underline{\frac{4}{3}}}$$

b) $f_{\frac{1}{2}}(x) = -\frac{3}{8}x^2 + \frac{1}{2}x + \frac{1}{2}$

$$A_f = \int_0^2 f_{\frac{1}{2}}(x)\,dx = \left[-\frac{1}{8}x^3 + \frac{1}{4}x^2 + \frac{1}{2}x\right]_0^2 = \underline{\underline{1}} = \frac{1}{2}A_\Delta \checkmark$$

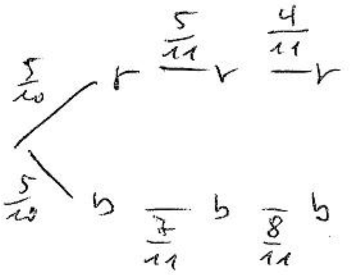
2. U_1: 5r / 5b
U_2: 4r / 6b
U_3: 3r / 7b
U_4: 0 / 0

a)



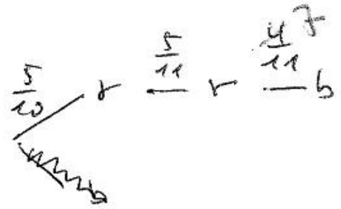
$$\left.\begin{array}{l}\frac{5}{10}\cdot\frac{4}{10}\\ \frac{5}{10}\cdot\frac{6}{10}\cdot\frac{1}{2}\\ \frac{5}{10}\cdot\frac{4}{10}\cdot\frac{1}{2}\end{array}\right\}\ \frac{90}{200} = \underline{\underline{45\,\%}}$$

b) (1)



$$\left.\begin{array}{l}\frac{5\cdot 5\cdot 4}{10\cdot 11\cdot 11}\\ \frac{5\cdot 7\cdot 8}{10\cdot 11\cdot 11}\end{array}\right\}\ \frac{380}{1210} = \underline{\underline{31{,}4\,\%}}$$

(2)



$$\frac{5\cdot 5\cdot \cancel{4}\,7}{10\cdot 11\cdot 11} = \frac{\overset{175}{\cancel{100}}}{1210} = \underline{\underline{\overset{14{,}5\,\%}{\cancel{8{,}26}\,\%}}}$$

3. $f(x) = a\sin x + \sin 2x \qquad [0; 2\pi[$ | Mass H 14

$f'(x) = a\cos x + 2\cos 2x$

$f'(\frac{\pi}{3}) = a\cdot\cos\frac{\pi}{3} + 2\cos(\frac{2\pi}{3}) = 0$

$a\frac{1}{2} + 2\cdot(-\frac{1}{2}) = 0$

$\underline{a = 2}$

$f(x) = 2\cdot\sin x + \sin 2x$

$f'(x) = 2\cos x + 2\cos 2x$

$f''(x) = -2\sin x - 4\sin 2x$

NST. $2\sin x + 2\sin x\cos x = 0$

$\underbrace{\sin x}\,(\underbrace{1+\cos x}) = 0$

$x = 0 + z\cdot 2\pi \qquad x = \pi + 2\cdot\pi$

$x = \pi + z\cdot 2\pi$

$\mathbb{L} = \{0; \pi\}$.

Ext. $\cos x + \cos 2x = 0$

$\cos x + 2\cos^2 x - 1 = 0$

$2\cos^2 x + \cos x - 1 = 0$

$\cos x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$

$\cos x_1 = \frac{1}{2}$ | $\cos x_3 = -1$ doppelt, kein Vorzeichenwechsel, $\underline{\text{Terrassenpunkt } (\pi + z2\pi \mid 0)}$

$x_1 = \frac{\pi}{3} + z\cdot 2\pi$ | $x_3 = \pi + z\cdot 2\pi$

$x_2 = \frac{5}{3}\pi + z\cdot 2\pi$

$f(x_3) = 0$

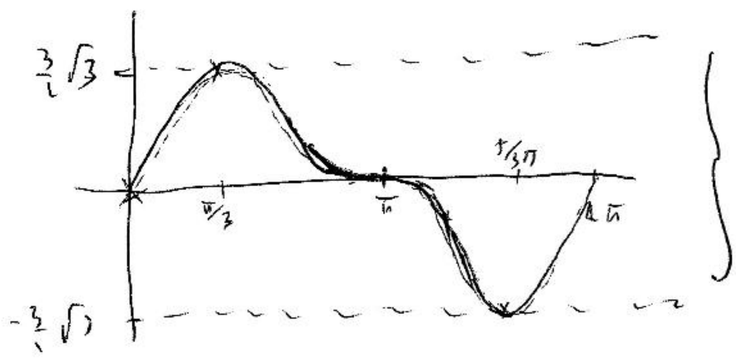
$f(x_1) = \frac{3}{2}\sqrt{3}$ — Max$(\frac{\pi}{3} \mid \frac{3}{2}\sqrt{3})$

$f(x_2) = -\frac{3}{2}\sqrt{3}$ — Tr.P.$(\pi \mid 0)$

Min$(\frac{5}{3}\pi \mid -\frac{3}{2}\sqrt{3})$

$f''(x_1) < 0 \Rightarrow \underline{\text{Max}(\frac{\pi}{3} + z\cdot 2\pi \mid \frac{3}{2}\sqrt{3})}$

$f''(x_2) > 0 \Rightarrow \underline{\text{Min}(\frac{5\pi}{3} + z2\pi \mid -\frac{3}{2}\sqrt{3})}$



$W = [-\frac{3}{2}\sqrt{3}; \frac{3}{2}\sqrt{3}]$

Pass H

4. $A(12|5)$ $B(12|11)$

$M: 3x-2y-32=0 : g$

$h \perp g$

$M_1: y=8 : x = \frac{2y+32}{3} = 16$

$\underline{M_1(16|8)}$

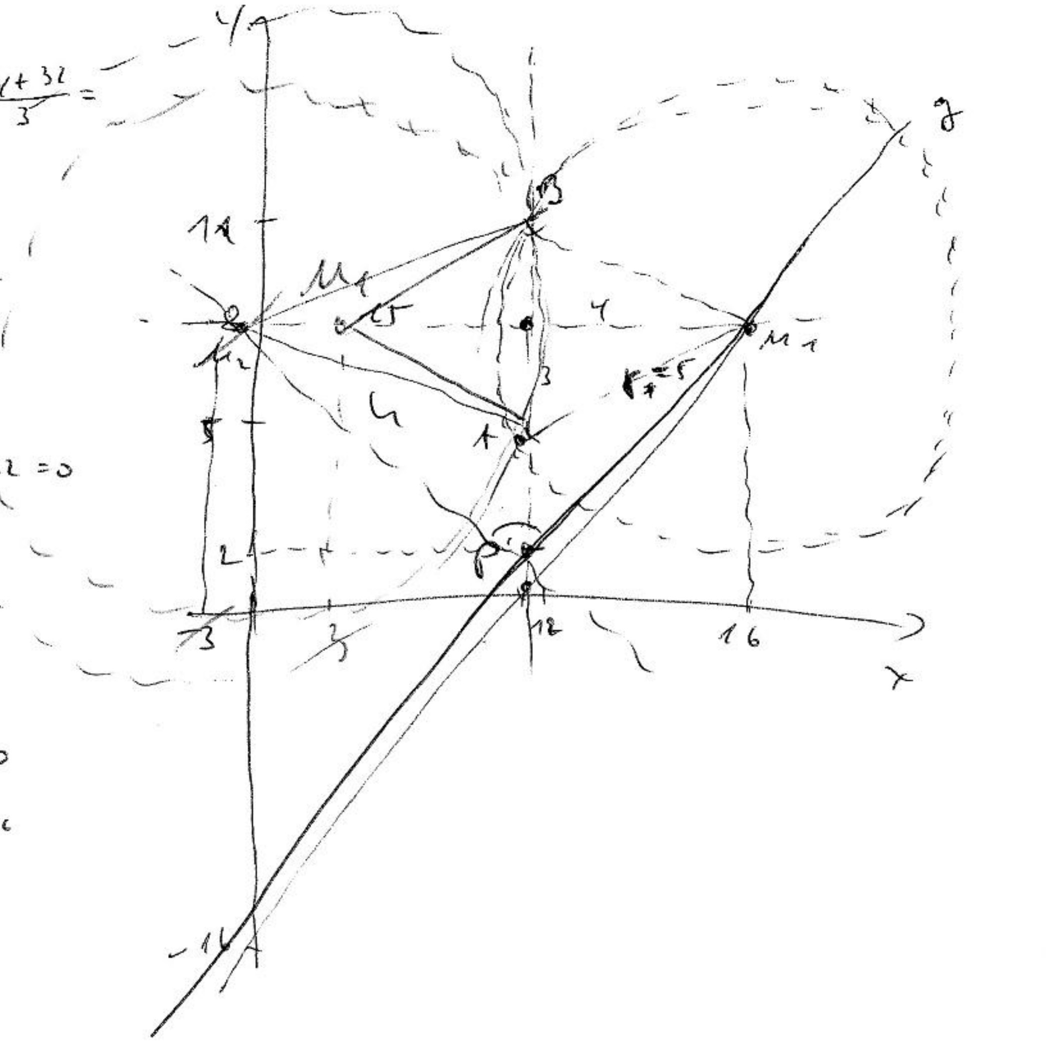
$\underline{r_1 = \sqrt{4^2+3^2} = 5}$

$AB: x = 12$

$AB \cap g: 3 \cdot 12 - 2y - 32 = 0$

$\underline{y = 2}$

$\underline{P(12|2)}$



$h \perp g:$

$h: 2x+3y+c=0$

$P(12|2): 2 \cdot 12 + 3 \cdot 2 + c = 0$

$\underline{c=-30}$

$\underline{h: 2x+3y-30=0}$

$h \cap y=8: \underline{x=3}$

$\underline{M_2(3|12)}$

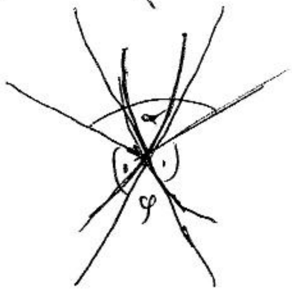
$\underline{r_2} = \sqrt{\cancel{15^2}+3^2} = \cancel{3\sqrt{26}}$ $\sqrt{90} = 3\sqrt{10}$ $\begin{pmatrix}-4\\-3\end{pmatrix}\begin{pmatrix}9\\-3\end{pmatrix}$

b) $\cos\alpha = \frac{\overline{M_1A} \cdot \overline{M_2A}}{M_1A \cdot M_2A} = \frac{\cancel{\begin{pmatrix}15\\-3\end{pmatrix}} \cancel{\begin{pmatrix}-4\\-3\end{pmatrix}}}{\sqrt{334} \cdot 5} = \frac{-60+9}{\cancel{\sqrt{334}} \cdot 5} = \frac{\cancel{-51}}{\cancel{\sqrt{334}} \cdot 5}$ -27 90

$\underline{\alpha = \cancel{131,8°}}$ $124,7°$

$\varphi = 360 - 2 \cdot 90 - \alpha$

$\varphi = \cancel{48,2°}$ $55,3°$

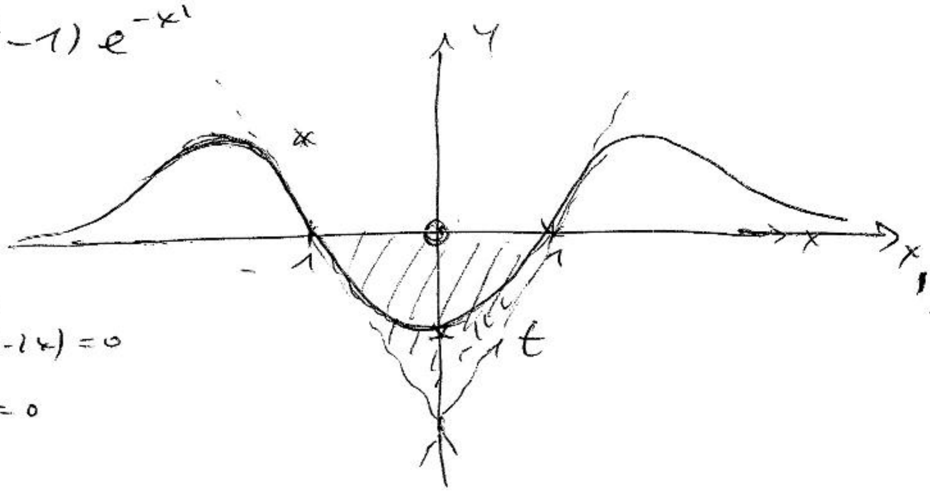


5. $f_k(x) = (x^2 - k^2)e^{-x^2} \qquad k > 0$ Pass H1

$k = 1: \quad f_1(x) = (x^2 - 1)e^{-x^2}$

NST: $\underbrace{(x^2-1)}e^{-x^2} = 0$

$\underline{x = \pm 1}$



$f'(x) = 2x e^{-x^2} + (x^2-1)e^{-x^2}(-2x) = 0$

$\underbrace{2x}e^{-x^2}\underbrace{(1-(x^2-1))} = 0$

$\underline{x = 0} \qquad x = \pm\sqrt{2}$

$\underline{y = e^{-2}}$

$f_k'(x) = 2x e^{-x^2} + (x^2 - k^2)e^{-k^2}(-2x)$

$= 2x e^{-x^2}(1 - (x^2 - k^2))$

$= 2x e^{-x^2}(1 + k^2 - x^2)$

$f_k(x) = 0 = \underbrace{(x^2 - k^2)}e^{-x^2}$

$\underline{x = \pm k}$

$f'(k) = 2k e^{-k^2}(1 + k^2 - k^2) = 2k e^{-k^2}$

$t: \quad y = 2k e^{-k^2}(x - k) + 0$

$t: \quad y = 2k e^{-k^2}x \underline{- 2k^2 e^{-k^2}}$

$\underline{N(\pm k | 0)}$

$f_k'(x) = 0$

$1 + k^2 - x^2 = 0$

$x = \pm\sqrt{1+k^2}$

$\underline{H(\pm\sqrt{1+k^2} | e^{-k^2-1})}$

$A_\triangle = \underbrace{\tfrac{1}{2}}_{\frac{1}{2}} \cdot \underbrace{2k}_{g} \cdot \underbrace{2k^2 e^{-k^2}}_{h} = 2k^3 e^{-k^2}$

$A' = 6k^2 e^{-k^2} + 2k^3 e^{-k^2}(-2k)$

$= 6k^2 e^{-k^2} - 4k^4 e^{-k^2} = 0$

$\underbrace{k^2}e^{-k^2}\underbrace{(6 - 4k^2)} = 0$

$(\underline{k=0}) \qquad \underline{k = \pm\sqrt{\tfrac{6}{4}} = \pm\tfrac{1}{2}\sqrt{6}} \qquad \underline{k = +\tfrac{1}{2}\sqrt{6}}$

VZT

k	1	$\frac{1}{2}\sqrt{6}$	2
$A'(k)$	$+$	0	$-$

$\nearrow \rightarrow \searrow$

$\underline{\underline{Max}}$

Ränder: $\lim\limits_{k\to 0} A(k) = \underline{0}$

$\lim\limits_{k\to\infty} \underbrace{2k^3}_{\infty}\underbrace{e^{-k^2}}_{0} = \underline{0}$

da Exp. [illegible]