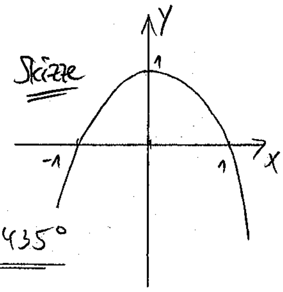
Passerelle, Herbst 2018

① a) N(1/0), N(−1/0) und P(0/1).

Skizze



$$\int_0^1 f(x)\,dx = \left[x - \frac{x^3}{3}\right]_0^1 = \frac{2}{3} \quad ; \quad A = 2\cdot\frac{2}{3} = \underline{\underline{\frac{4}{3}}}$$

$$m_1 = f'(1) = -2 = \tan\alpha_1 \rightarrow \underline{\underline{\alpha_1 = 116.565^\circ}}, \quad \underline{\underline{\alpha_2 = 63.435^\circ}}$$

b1) Nullstellen: $d - d^3 \cdot x^2 = 0 \rightarrow x = \pm\frac{1}{d}$

$$\int_0^{1/d} f(x)\,dx = \left[dx - \frac{d^3}{3}x^3\right]_0^{1/d} = 1 - \frac{1}{3} = \frac{2}{3} \quad ; \quad A = 2\cdot\frac{2}{3} = \frac{4}{3}$$

unabhängig von d.

b2) $f'(x) = -2d^3\cdot x \quad ; \quad f'\left(\frac{1}{d}\right) = -2d^2 = m_1$

$$f'\left(-\frac{1}{d}\right) = 2d^2 = m_2$$

Bedingung $m_1 \cdot m_2 = -1 \rightarrow -4d^4 = -1 \rightarrow d = \sqrt[4]{\frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

die Basis des Dreiecks hat die Länge $2\cdot\frac{2}{\sqrt{2}} = 2\sqrt{2}$ (Abstand der Nullstellen)

die Schenkel haben die Länge $\frac{2}{\sqrt{2}}\cdot\sqrt{2} = 2$

Flächeninhalt des Dreiecks $A = \frac{1}{2}\cdot 2^2 = \underline{\underline{2}}$

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② a1) $P = 0.7^3 = \underline{\underline{34.3\,\%}}$

a2) $P = 0.3^3 + 0.3^2\cdot 0.7\cdot 3 = \underline{\underline{21.6\,\%}}$

a3) $1 - 0.7^n \geq 0.9 \rightarrow 0.7^n \leq 0.1 \rightarrow n \geq \frac{\ln 0.1}{\ln 0.7} \approx 6.5 \rightarrow \underline{\underline{n \geq 7}}$

b1) $P_1 = 0.7^3\cdot 0.8^3 = \underline{\underline{17.56\%}}$

b2) $P_2 = 0.2 + 0.8\cdot 0.7\cdot 0.2 + 0.8^2\cdot 0.7^2\cdot 0.2 = \underline{\underline{37.47\%}}$

b3) $P_3 = 1 - P_1 - P_2 = \underline{\underline{44.97\%}}$ $(= 0.8\cdot 0.3 + 0.8^2\cdot 0.7\cdot 0.3 + 0.8^3\cdot 0.7^2\cdot 0.3)$

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③ a) $\vec{AB} = \begin{pmatrix}-x\\3\end{pmatrix}, \quad \vec{AC} = \begin{pmatrix}-x\\-2\end{pmatrix}$

$$\vec{AB}\cdot\vec{AC} = x^2 - 6 = \sqrt{x^2+9}\cdot\sqrt{x^2+4}\cdot\frac{\sqrt{3}}{2}$$

$$x^4 - 12x^2 + 36 = (x^2+9)(x^2+4)\cdot\frac{3}{4}$$

$$x^4 - 12x^2 + 36 = \frac{3}{4}(x^4 + 13x^2 + 36)$$

$$\frac{1}{3}x^4 - 29x^2 + 12 = 0 \rightarrow x^4 - 87x^2 + 36 = 0$$

$$x^2 = u \rightarrow u = \frac{87 \pm \sqrt{7425}}{2} = \frac{87 \pm 15\cdot\sqrt{33}}{2} = \begin{cases} 86.584 \\ 0.4158 \end{cases}$$

$\rightarrow$ ($\underline{\underline{x = \pm 0.6448}}$) $\quad \underline{\underline{x = \pm 9.3051}}$

Probe

③ b) $A(x) = x \cdot \sqrt{1-\frac{x^2}{4}} = \sqrt{x^2 - \frac{x^4}{4}}$

$A'(x) = \dfrac{2x - x^3}{2\sqrt{x^2 - \frac{x^4}{4}}}$; setze $A'(x) = 0 \rightarrow 2x - x^3 = 0$

$x = 0, \quad x = \pm\sqrt{2}$

A maximal für $\underline{\underline{x = \sqrt{2}}}$; $A_{max} = \sqrt{2-1} = \underline{\underline{1}}$

Nachweis, dass tatsächlich maximal:

x		0		$\sqrt{2}$	
$A'(x)$			+	0	−
$A(x)$			↗	H	↘

Randwerte: $A(0) = 0$
$A(2) = 0$.

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④ a) $x = 2-2y$ einsetzen $\rightarrow$ $(-2y-1)^2 + (y-2)^2 = \frac{25}{4}$

$5y^2 + 5 = \frac{25}{4}$

$y^2 = \frac{1}{4} \rightarrow y = \frac{1}{2}, \quad x = 2 - 2\cdot\frac{1}{2} = 1$

$y = -\frac{1}{2}, \quad x = 2 + 2\cdot\frac{1}{2} = 3$

$\rightarrow \underline{\underline{S_1(1/0.5)}}$

$\underline{\underline{S_2(3/-0.5)}}$

b) <u>Lösungsweg 1</u>

$K: (x-3)^2 + (y-v)^2 = 20$

$x = 8-2y$ einsetzen $\rightarrow (5-2y)^2 + (y-v)^2 = 20$

$5y^2 + (-2v-20)\cdot y + v^2 + 5 = 0$

Bedingung: genau eine Lösung, dh. Diskriminante $D = 0$.

$D = (-2v-20)^2 - 20\cdot(v^2+5) = -16v^2 + 80v + 300 = 0$

$4v^2 - 20v - 75 = 0$

$v = \dfrac{20 \pm 40}{8} = \begin{cases} 7.5 \\ -2.5 \end{cases}$

$\rightarrow \underline{\underline{K_1: (x-3)^2 + (y-7.5)^2 = 20}}$

$\underline{\underline{K_2: (x-3)^2 + (y+2.5)^2 = 20}}$

<u>Lösungsweg 2</u>

Bedingung: $d(M, g) = R = \sqrt{20}$

g in Hess'scher Normalform: $\dfrac{x + 2y - 8}{\sqrt{5}} = 0$

$M(3/v)$

$d(M, g) = \left| \dfrac{3 + 2v - 8}{\sqrt{5}} \right| = \sqrt{20}$

1. Mögl. $2v - 5 = \sqrt{100} = 10 \rightarrow v = 7.5$

2. Mögl. $2v - 5 = -10 \rightarrow v = -2.5$