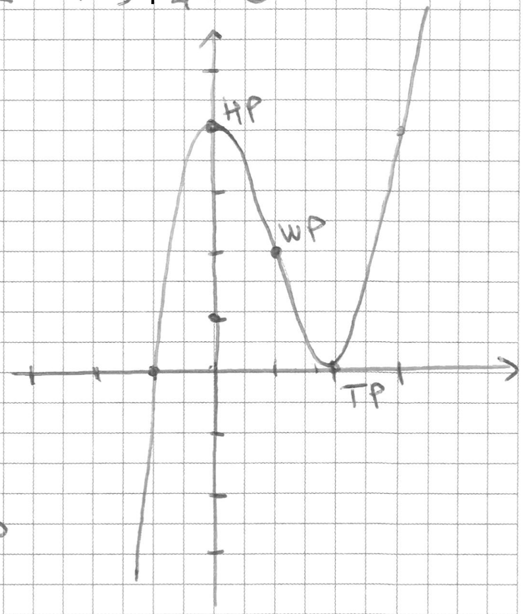
AKAD.CH

1) a) $f(-1) = (-1)^3 - 3(-1)^2 + 4 = -1-3+4 = 0$

b) $f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$



EXTREMA

$f'(x) = 0 \rightarrow 3x(x-2) = 0$

$x_1 = 0 \qquad x_2 = 2$

$y_1 = f(0) = 4 \qquad y_2 = f(2) = 0$

VZW $\rightarrow$ $(0,4)$ HP $\quad (2,0)$ TP

WENDEPUNKT

$f''(x) = 0 \rightarrow 6(x-1) = 0 \quad x = 1 \rightarrow y = f(1) = 2 \quad (1,2)$ WP

(KONTROLLE $f'''(x) = 6 \neq 0$)

c) $\int_{-1}^{2} f(x)\,dx = \frac{1}{4}x^4 - x^3 + 4x \Big|_{-1}^{2} =$

$= \left(\frac{1}{4}\cdot 2^4 - 2^3 + 4\cdot 2\right) - \left(\frac{1}{4}(-1)^4 - (-1)^3 + 4(-1)\right) = 4 - \left(-\frac{11}{4}\right) = \frac{27}{4}$

d) $m_T = f'(1) = 3\cdot 1^2 - 6\cdot 1 = -3$

$t: y = -3(x-1) + 2 \rightarrow 0 = -3(x-1) + 2$

Die Tangente t schneidet die x-Achse a.d.S. $x = \frac{5}{3}$

---

2) a) $P(WW) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

b) $P(SSSS) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

c) $P(\#S \geq 1) = 1 - P(\#W = 6) = 1 - \left(\frac{1}{2}\right)^6 = \frac{63}{64}$

AKAD

d) $p(XX) = p(ww) + p(SS) = \frac{2}{3}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{1}{3} = \frac{5}{9}$

$p(XY) = p(wS) + p(Sw) = 2\cdot\frac{2}{3}\cdot\frac{1}{3} = \frac{4}{9}$

Es ist wahrscheinlicher 2 gleichfarbige Kugel zu ziehen

e) $P(WS) + P(SW) = \frac{5}{5+5+R}\cdot\frac{5}{4+5+R} + \frac{5}{5+5+R}\cdot\frac{5}{5+4+R} = \frac{5}{119}$

$\frac{50}{(10+R)(9+R)} = \frac{5}{119} \to R^2 + 19R - 1100 = 0$

$(R-25)(R+44) = 0 \to R_1 = 25 \quad R_2 = -44$

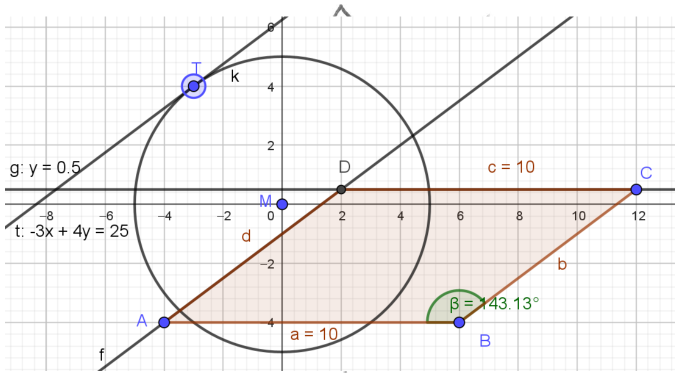
Es sind 25 rote Kugeln.

3) a) $r = \overline{MT} = \sqrt{3^2+4^2} = 5 \to K: x^2 + y^2 = 5^2$

$t \perp \vec{MT} \to \vec{v_t} = \begin{pmatrix}4\\3\end{pmatrix}$

$t: \vec{r} = \vec{r_T} + s\vec{v_t}$

$\vec{r} = \begin{pmatrix}-3\\4\end{pmatrix} + s\begin{pmatrix}4\\3\end{pmatrix}$



b) $|AB| = 10 \to B = (6|-4)$

$g_{BC}: \vec{r} = \begin{pmatrix}6\\-4\end{pmatrix} + t\begin{pmatrix}4\\3\end{pmatrix}$ (parallel zu t durch B)

$C = g_{BC} \cap \{y = \frac{1}{2}\}$

$\to \begin{cases} x = 6 + 4t = 12 \\ y = -4 + 3t = \frac{1}{2} \to t = 1{,}5 \end{cases}$

$\to C = (12|0{,}5)$

$D = (2|0{,}5)$

c) β ist Stumpf.

$\cos\beta = \frac{\vec{BA}\cdot\vec{BC}}{|BA|\cdot|BC|} = \frac{\begin{pmatrix}-10\\0\end{pmatrix}\cdot\begin{pmatrix}28\\21\end{pmatrix}}{10\cdot\sqrt{28^2+21^2}} = \frac{-280}{10\cdot 35} = -\frac{4}{5}$

$\beta = \cos^{-1}\left(-\frac{4}{5}\right) \approx 143°$

AKAD.CH

4) a) $\vec{AB} = \begin{pmatrix} -4 \\ 6 \\ -12 \end{pmatrix}$ $\quad \vec{AC} = \begin{pmatrix} 2 \\ 12 \\ -4 \end{pmatrix}$ $\quad \vec{n} = \vec{AB} \times \vec{AC} = \begin{pmatrix} 120 \\ -40 \\ -60 \end{pmatrix}$

$$E: \vec{r} = \vec{r}_A + s\vec{AB} + t\vec{AC}$$

$$\vec{r} = \begin{pmatrix} 5 \\ -6 \\ 17 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -12 \end{pmatrix} + t\begin{pmatrix} 2 \\ 12 \\ -4 \end{pmatrix}$$

Elimination

I $x = 5 - 4s + 2t$

II $y = -6 + 6s + 12t$

III $z = 17 - 12s - 4t$

$2\,\text{II} + \text{III}$ $\quad 2y + z = 5 + 20t$

$3\,\text{I} - \text{III}$ $\quad 3x - z = -2 + 10t$

$2y + z - 2(3x - z) = 5 - 2\cdot(-2) \rightarrow 6x - 2y - 3z + 9 = 0$

Normalenform

$$\vec{n} \cdot (\vec{r} - \vec{r}_A) = 0$$

$$\begin{pmatrix} 120 \\ -40 \\ -60 \end{pmatrix} \cdot \begin{pmatrix} x - 5 \\ y + 6 \\ z - 17 \end{pmatrix} = 0$$

$\downarrow$

b) $g_{OS}: \vec{r} = \vec{r}_0 + p\,\vec{OS}$

$$\vec{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + p\begin{pmatrix} 14 \\ -21 \\ 21 \end{pmatrix}$$

$g_{OS} \cap E \rightarrow \quad 6(0 + 14p) - 2(0 - 21p) - 3(0 + 21p) + 9 = 0$

$63p + 9 = 0 \rightarrow p = -\frac{1}{7}$

$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \frac{1}{7}\begin{pmatrix} 14 \\ -21 \\ 21 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ -3 \end{pmatrix} \rightarrow$ Der Schnittpunkt ist $(-2|3|-3)$

c) $|AB| = \sqrt{(-4)^2 + 6^2 (-12)^2} = 14 \rightarrow |CD| = 7$

$\vec{AB} \parallel \vec{CD}$ & $|AB| = 2|CD| \rightarrow \vec{CD} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \rightarrow$

$\rightarrow D = \text{"}C + \vec{CD}\text{"} = (9|3|19)$

d)

MIT KREUZPRODUKT

$A = \frac{1}{2}|\vec{AB} \times \vec{AC}| + \frac{1}{2}|\vec{AC} \times \vec{AD}|$

$= \frac{1}{2}\left|\begin{pmatrix} 120 \\ -40 \\ -60 \end{pmatrix}\right| + \frac{1}{2}\left|\begin{pmatrix} 60 \\ -20 \\ -30 \end{pmatrix}\right|$

$= \frac{1}{2}\cdot 140 + \frac{1}{2}\cdot 70 = 105$

OHNE KREUZPRODUKT

$A = m \cdot h$

$m = \frac{1}{2}(|AB| + |CD|) = \frac{21}{2}$

$h = |CH|$

mit $\vec{CH} \perp \vec{AB}$

$\rightarrow H = ??$

AKAD